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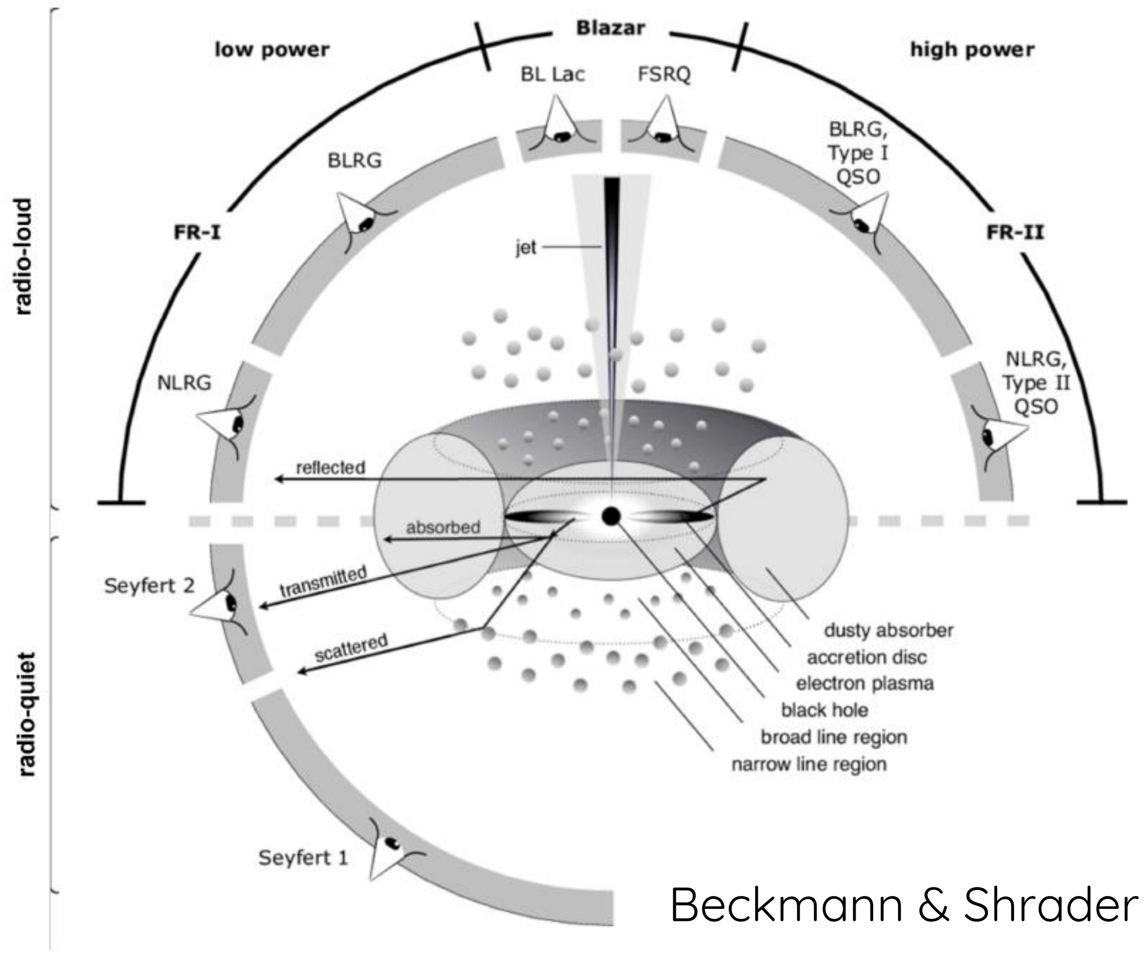
Exploring the Physics of Very-High-Energy Astroparticle Emission in Blazars through VLBI Observations

Florian Eppel

Julius-Maximilians-Universität Würzburg & MPIfR Bonn

Julius-Maximilians-UNIVERSITÄT WÜRZBURG

AGN Zoo



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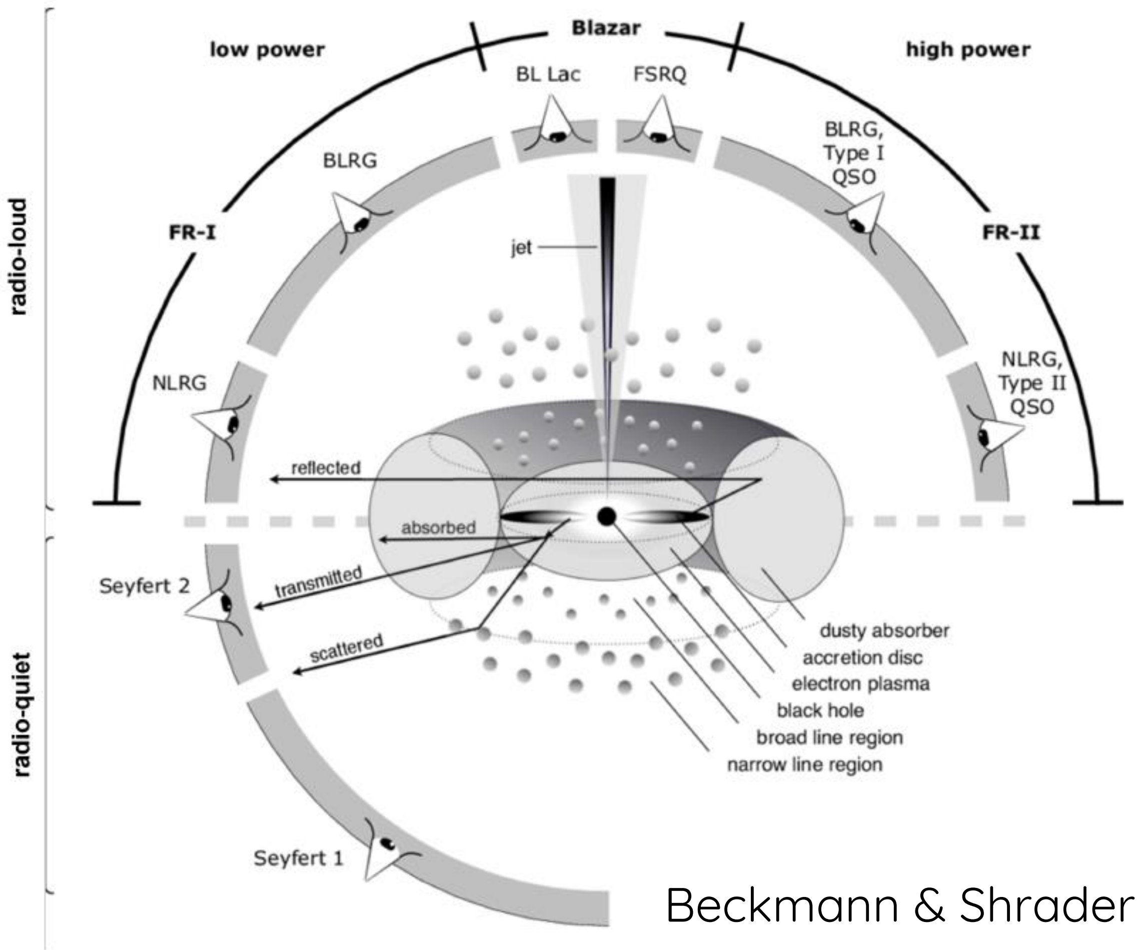

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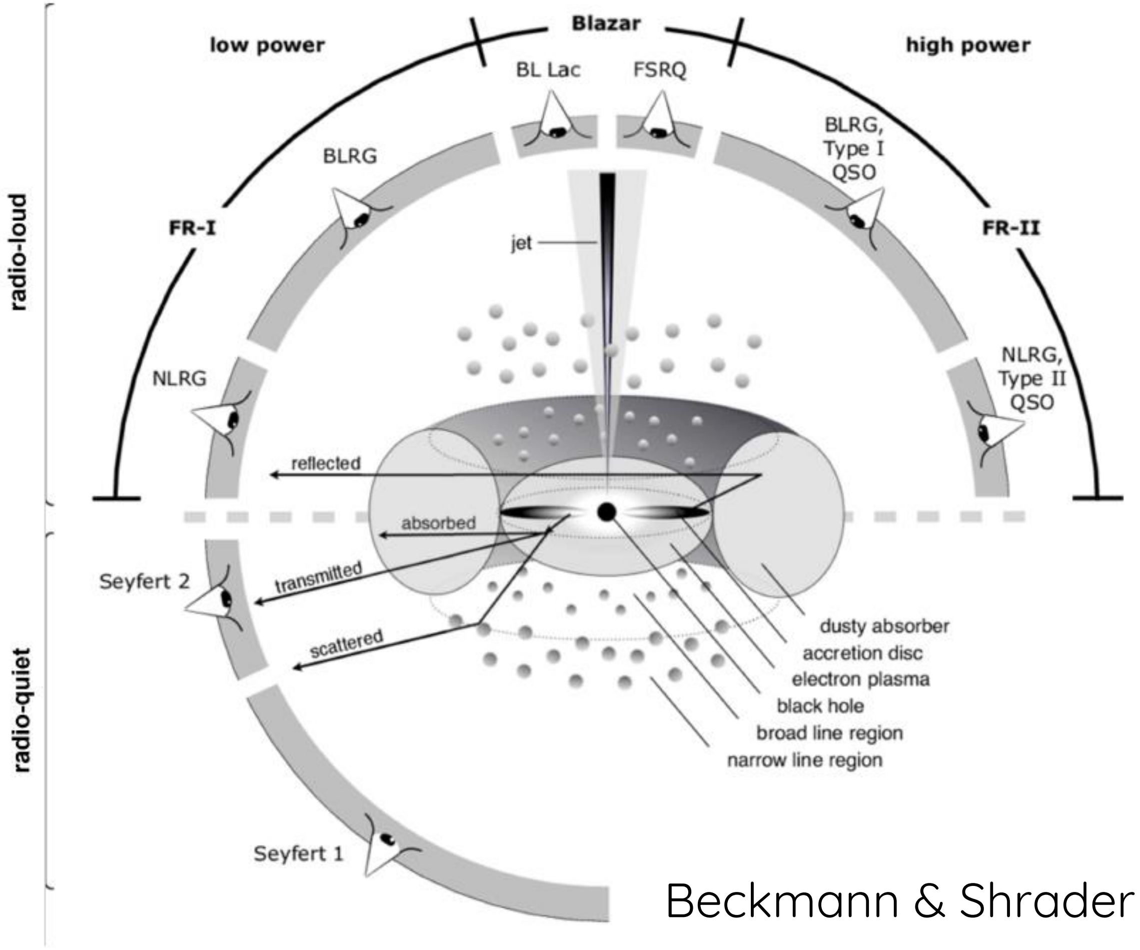

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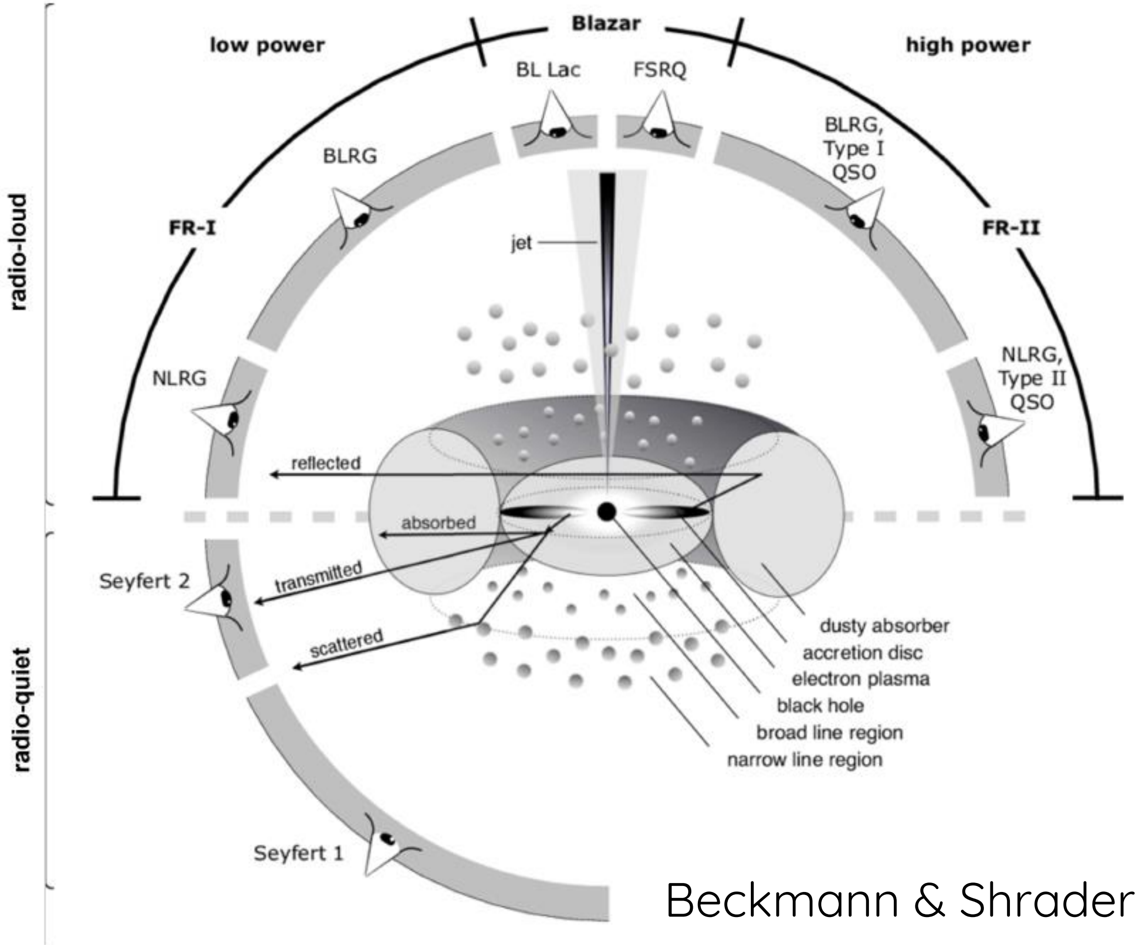

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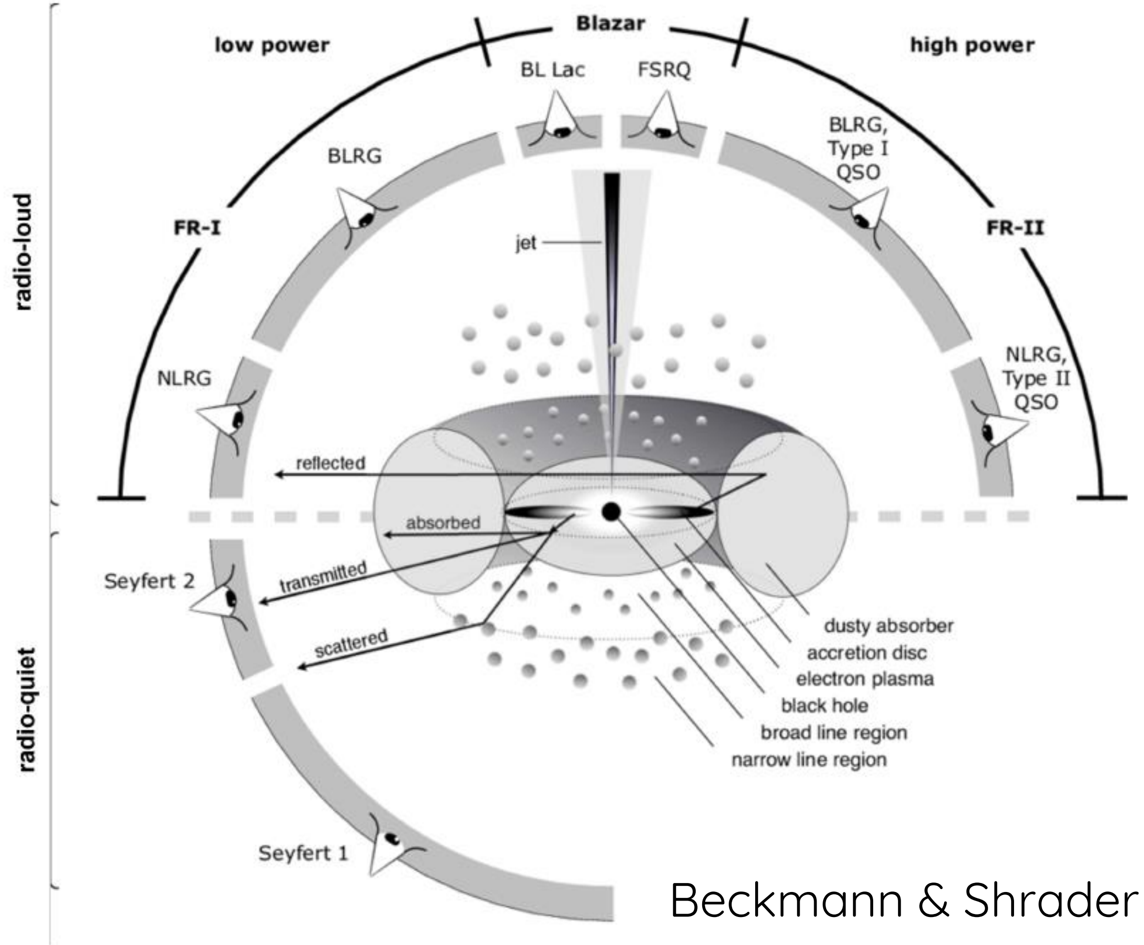

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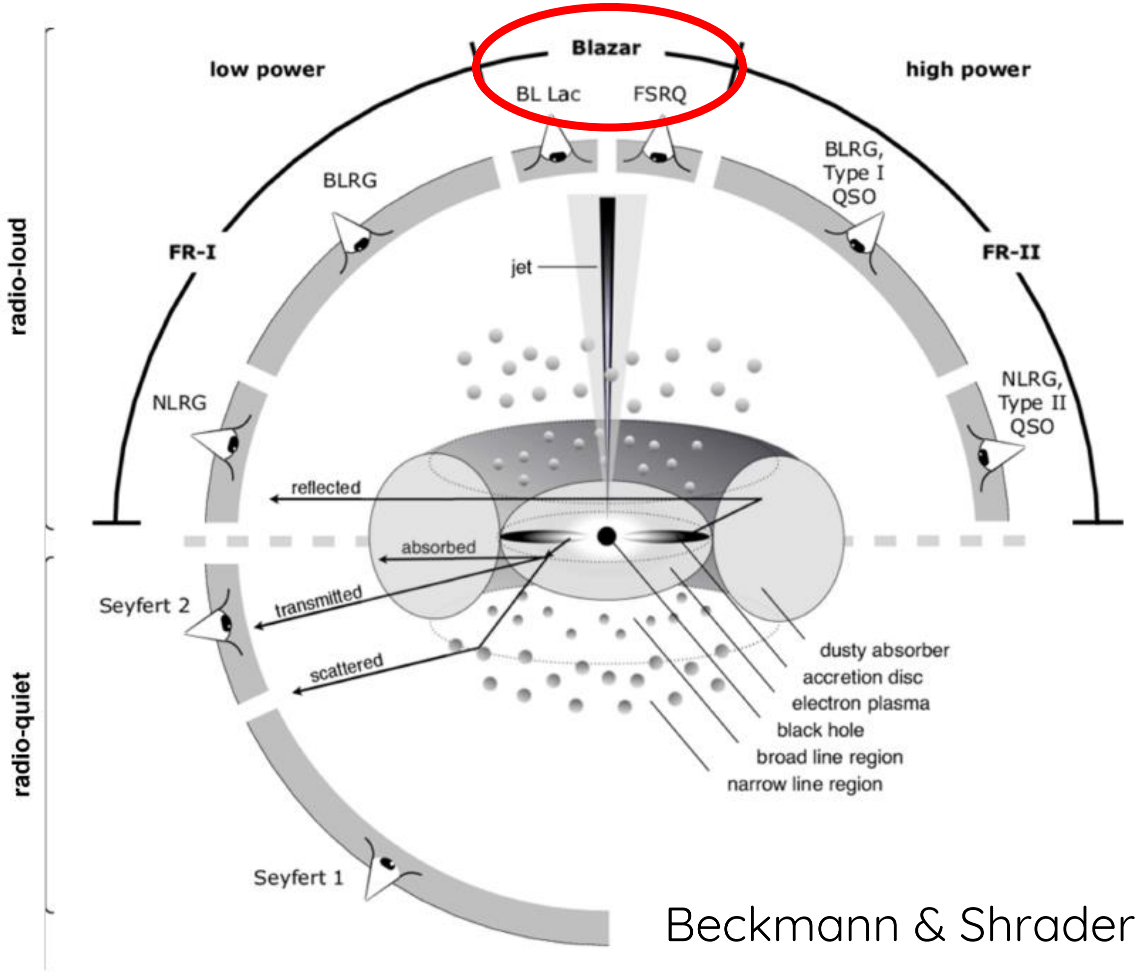

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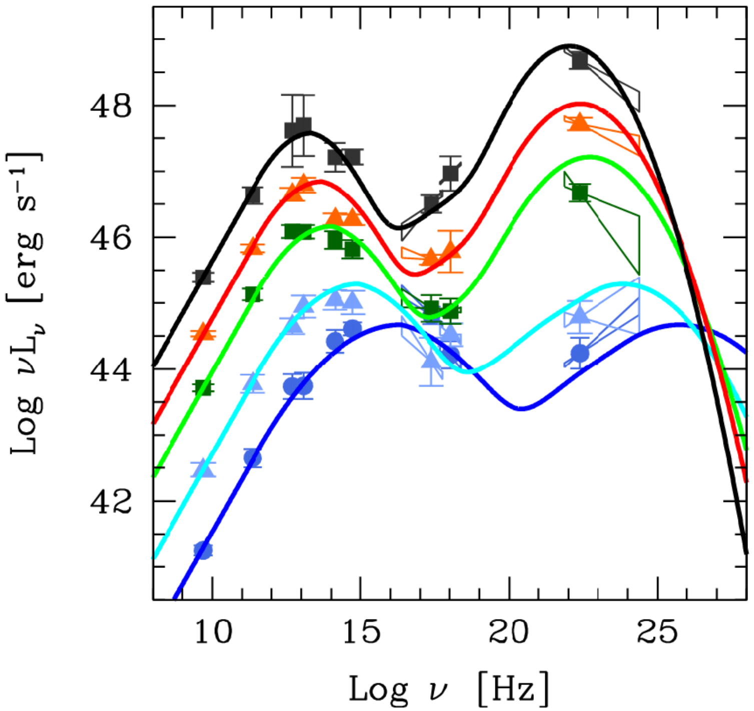

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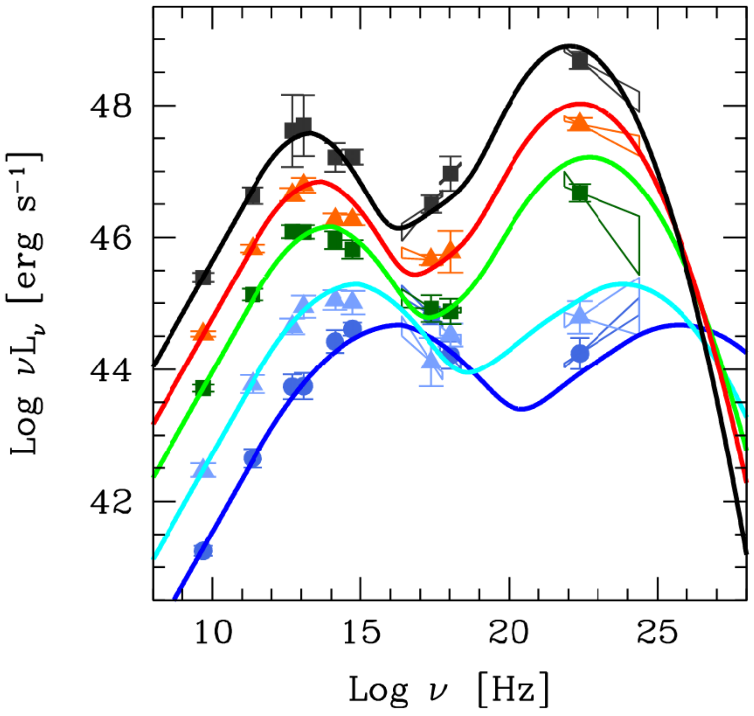

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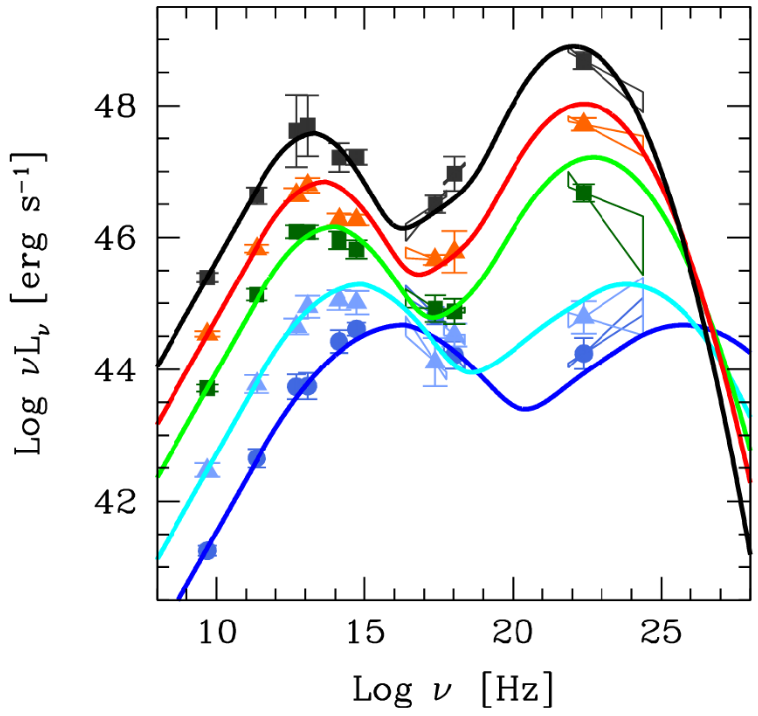

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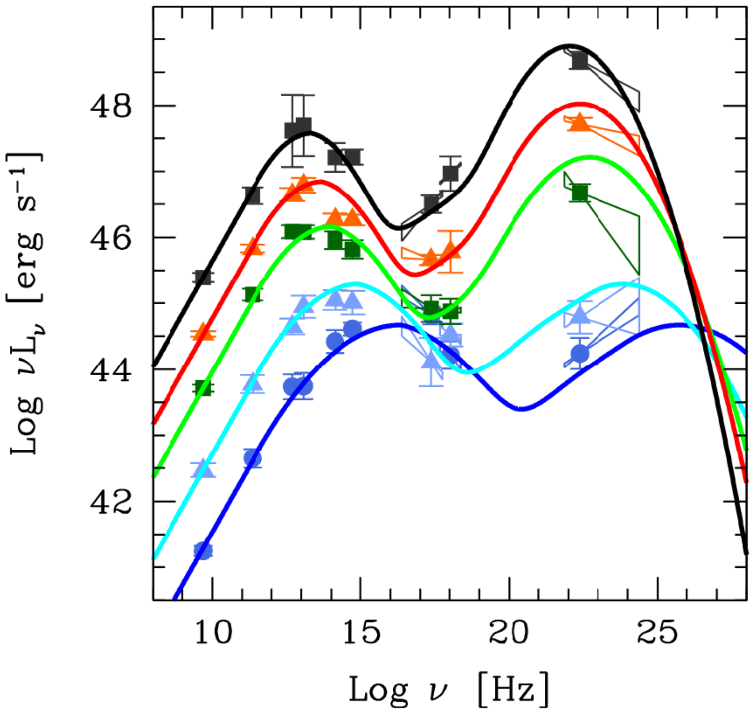
  - LBL ($\nu_{peak} < 10^{14}$ Hz)
  - IBL (10^{14} Hz $< \nu_{peak} < 10^{15}$ Hz)
  - HBL ($\nu_{peak} > 10^{15}$ Hz)
  - EHBL ($\nu_{peak} > 10^{17}$ Hz or other criteria)
- 81 TeV-detected (dominated by HBL)



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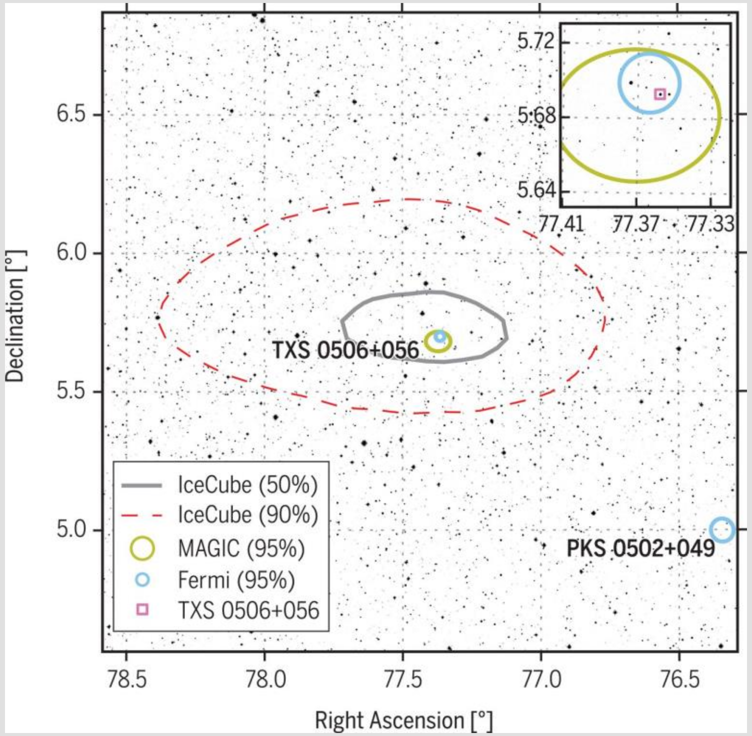
- Standing Recollimation Shocks (Hervet et al., 2019)

Blazars - A Source of High Energy Neutrinos?



Localization Region of IC170922A (IceCube Collaboration et al. 2018)

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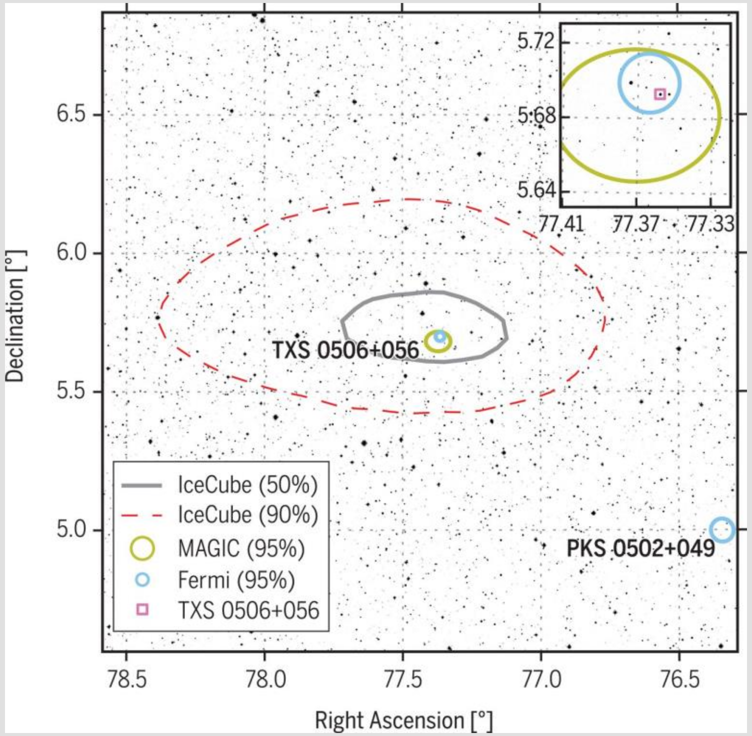

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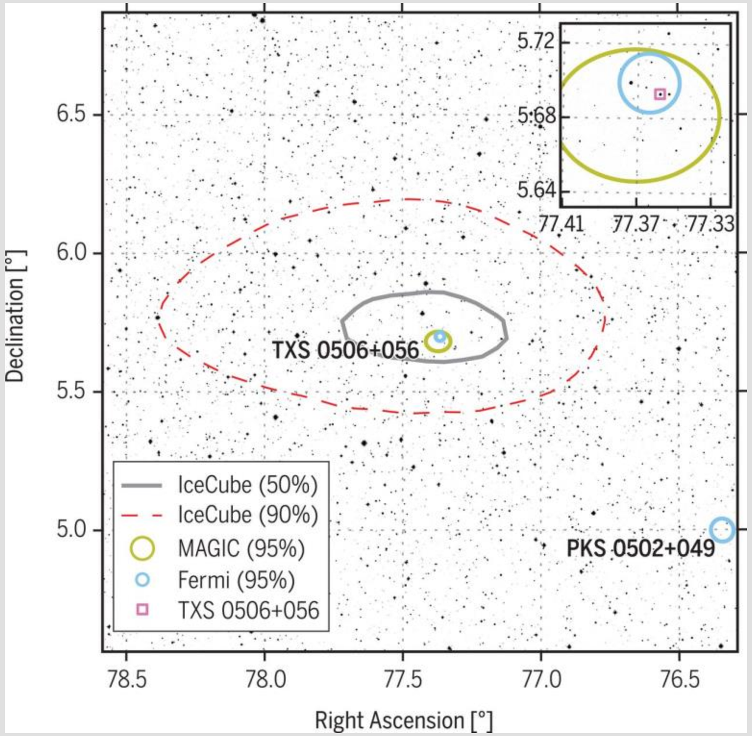

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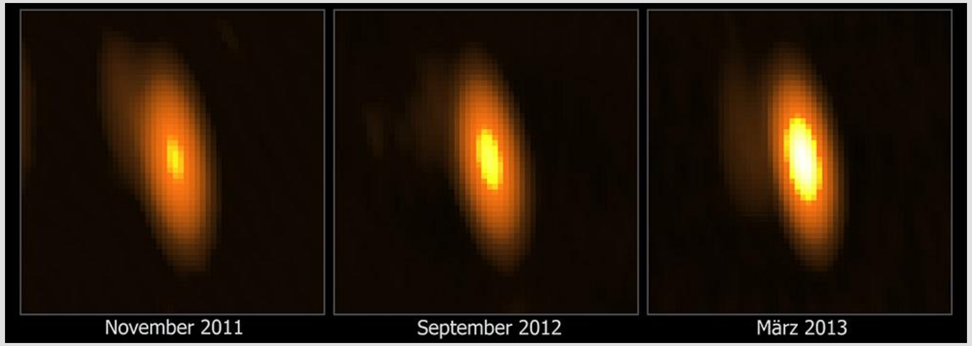
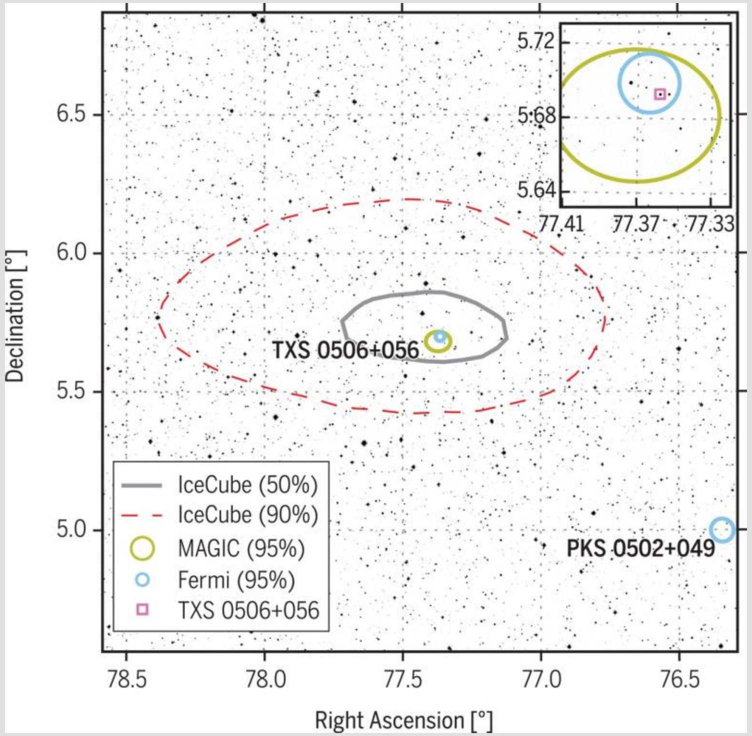
- Several studies suggest connection between radio-bright AGN and high-energy neutrino emission (Plavin et al. 2020 & 2021, Hovatta et al. 2020, Kadler et al. 2016)



Radio Images of PKS 1424-418 before and after a possibly coincident (2σ) Neutrino Event (Kadler et al. 2016)



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- With **VLBI**, scales down to the central engine and the jet base can be investigated, as well as the **jet-speed**, **jet geometry** and **magnetic-field** can be probed

Doppler Crisis & Neutrino Emission

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→ Need for more statistics and individual case-studies

TELAMON

- **T**eV **E**ffelsberg **L**ong-term **A**gn **MON**itoring

Effelsberg 100-m Telescope

Eppel et al. (2024, A&A, 684, A11)

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- High cadence observations at high radio frequencies
  - Every 2-4 weeks
  - From 5 GHz up to 44 GHz (45mm, 20mm, 14mm, 7mm)

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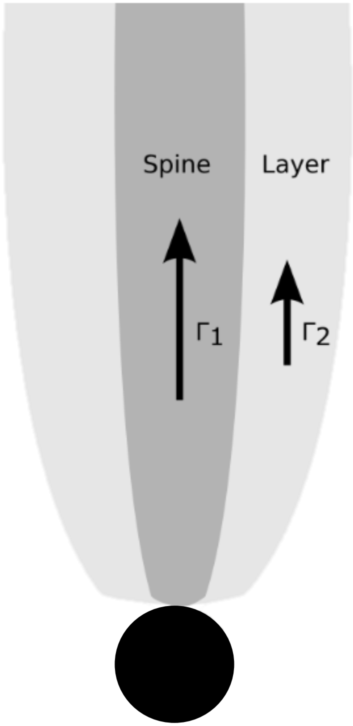
- **Doppler factor**, viewing angle and jet speed can be probed
- VLBI can constrain different models:
  - Limb-Brightening/Spine-Sheath Structure (Tavecchio et al. 2014, 2015)
  - Standing Recollimation Shocks (Kalashev et al. 2023)



Rieger & Levinson, 2018

Updates on ongoing projects

Doppler-Crisis Case Studies

- PG 1553+113: A supermassive binary black hole candidate in outburst

VLBI Probes of Neutrino-Candidate Blazars

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PG 1553+113: HBL and SMBBH

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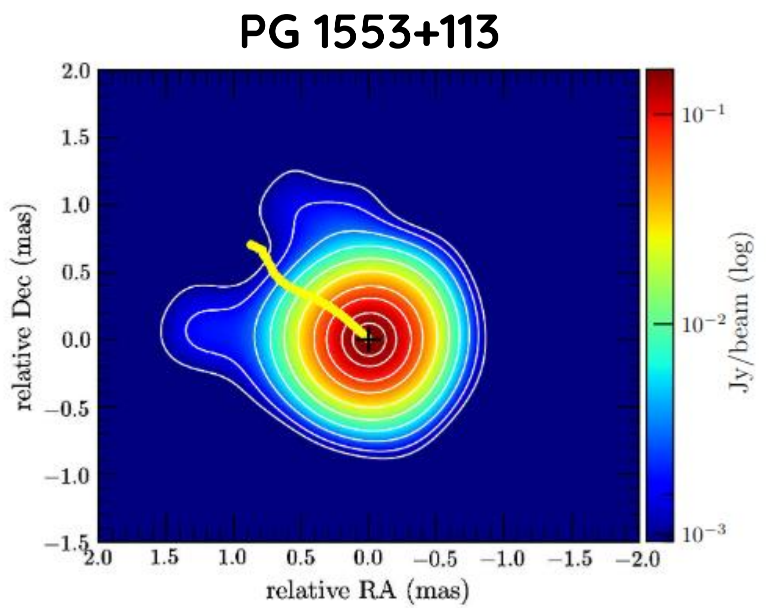

15 GHz VLBA image of PG 1553+113
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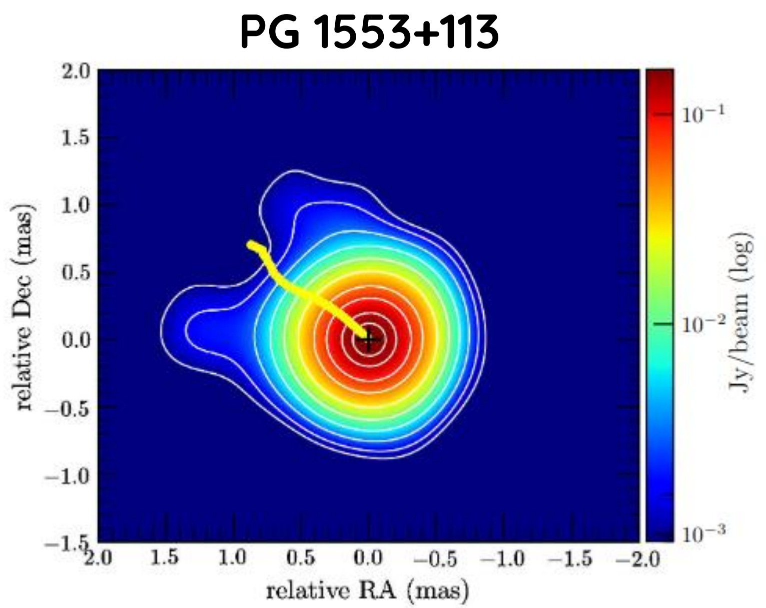
PG 1553+113: HBL and SMBBH



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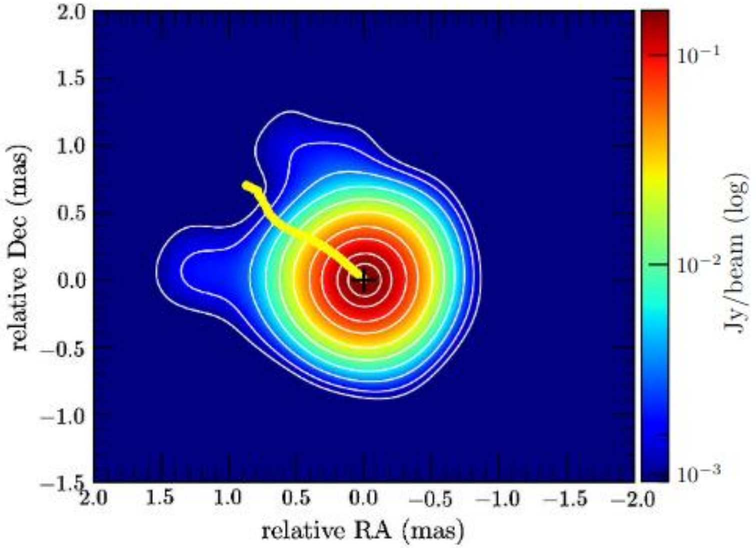

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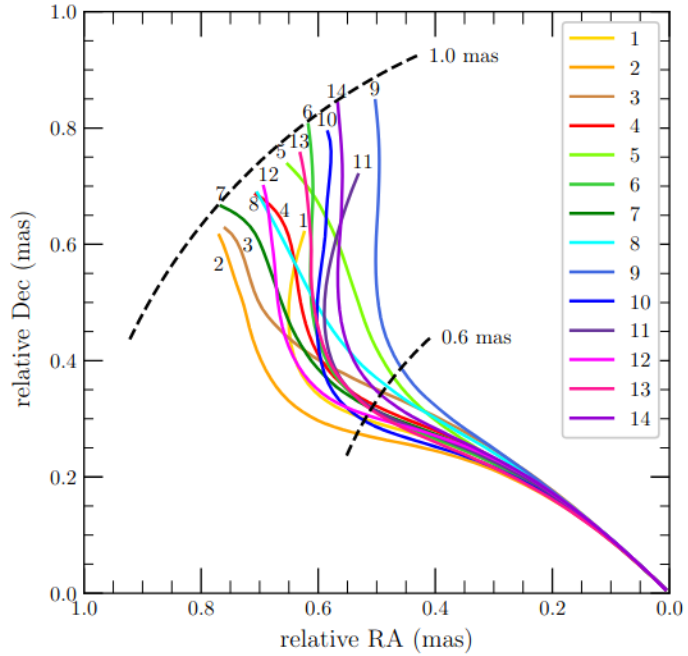
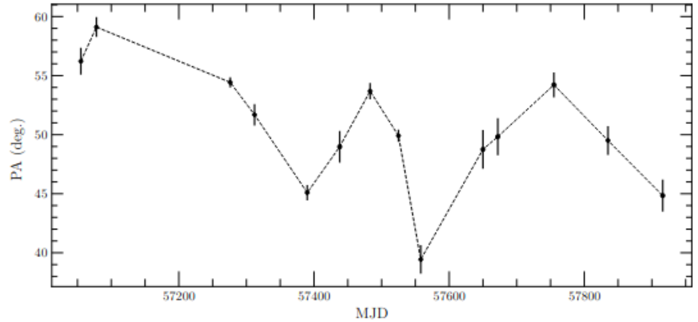
- Previous VLBI study by Lico et al. (2020) revealed possible jet wobbling and hints of limb-brightening





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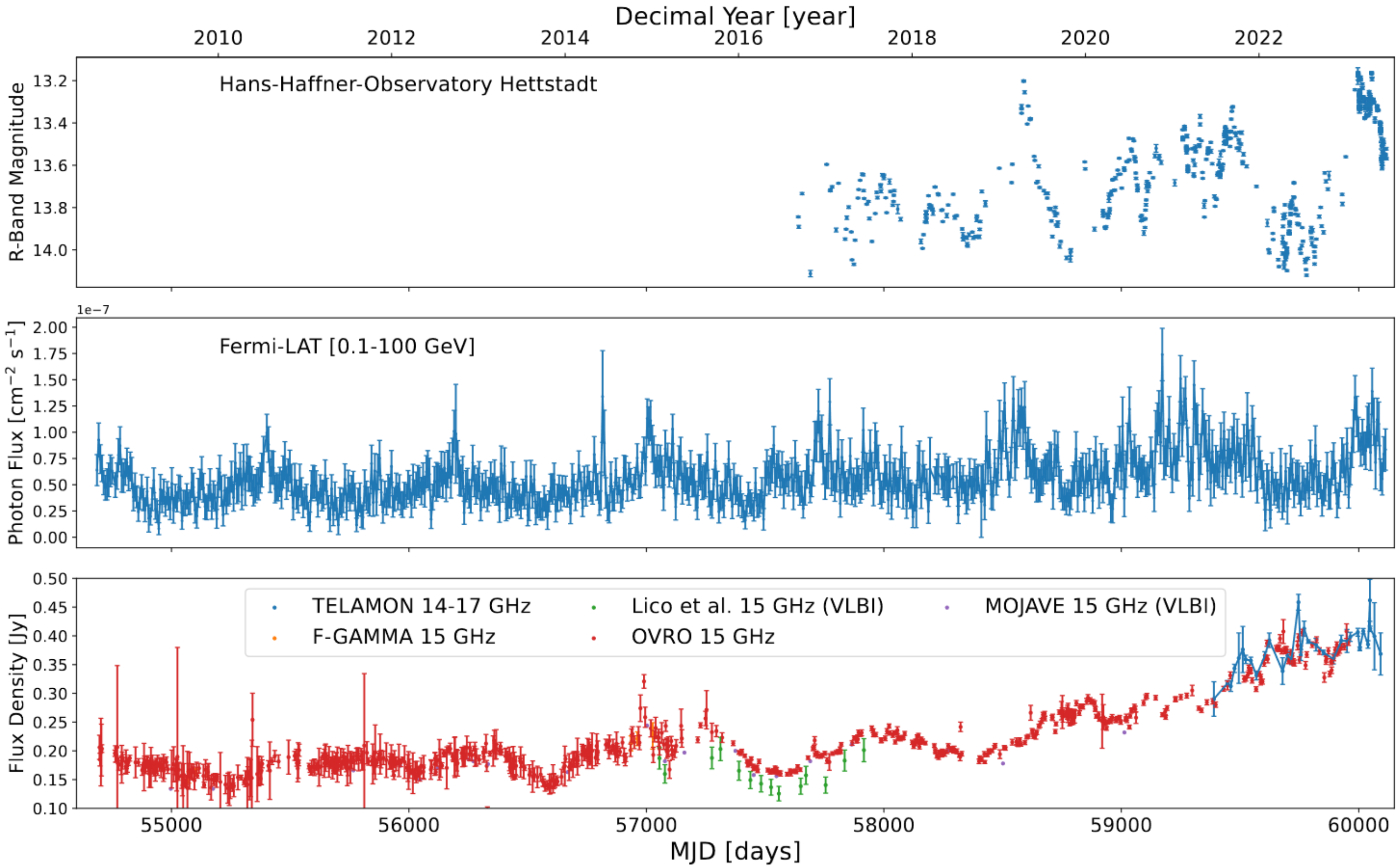

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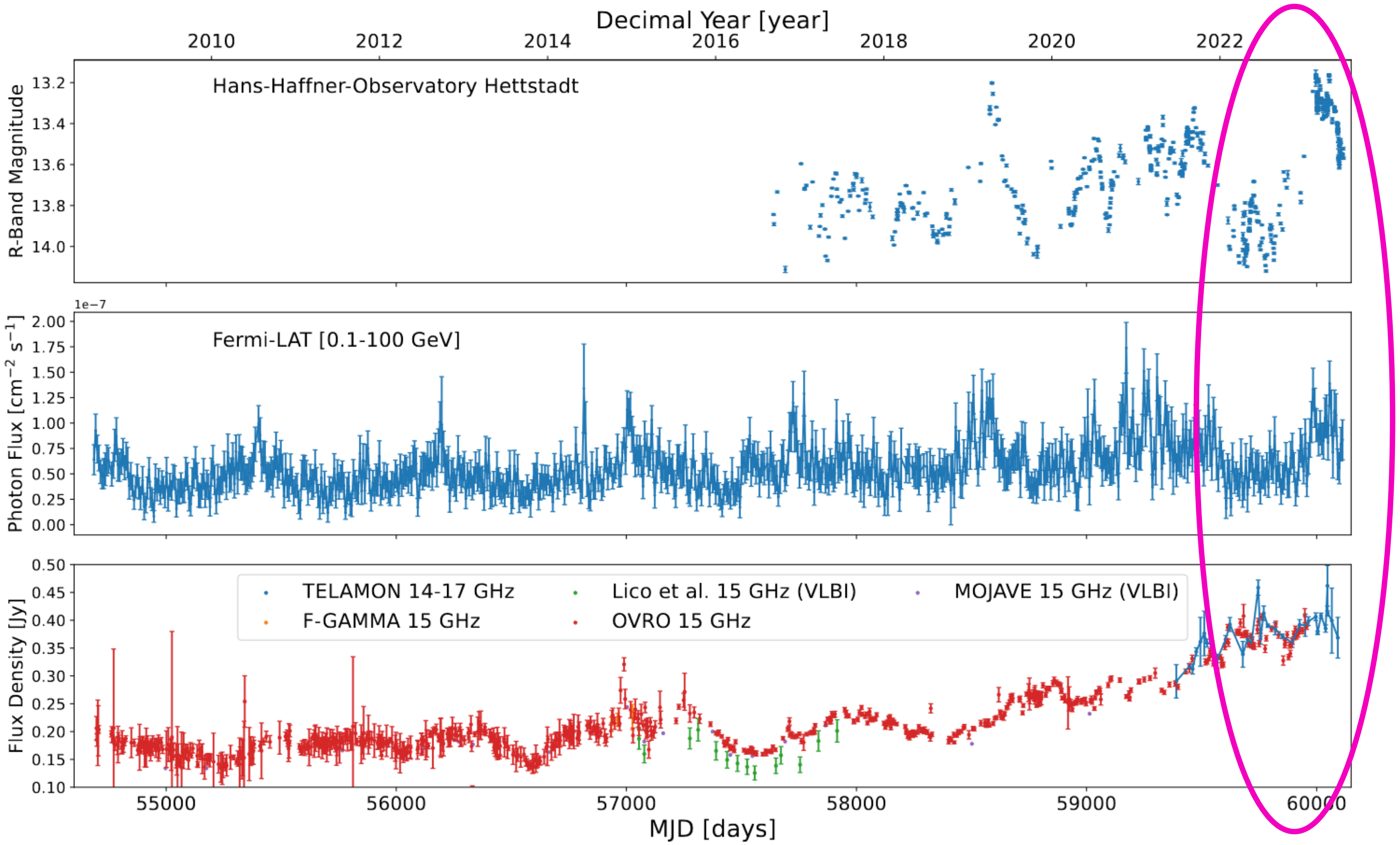

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- Triggered EHT observation with simultaneous MWL campaign

EHT

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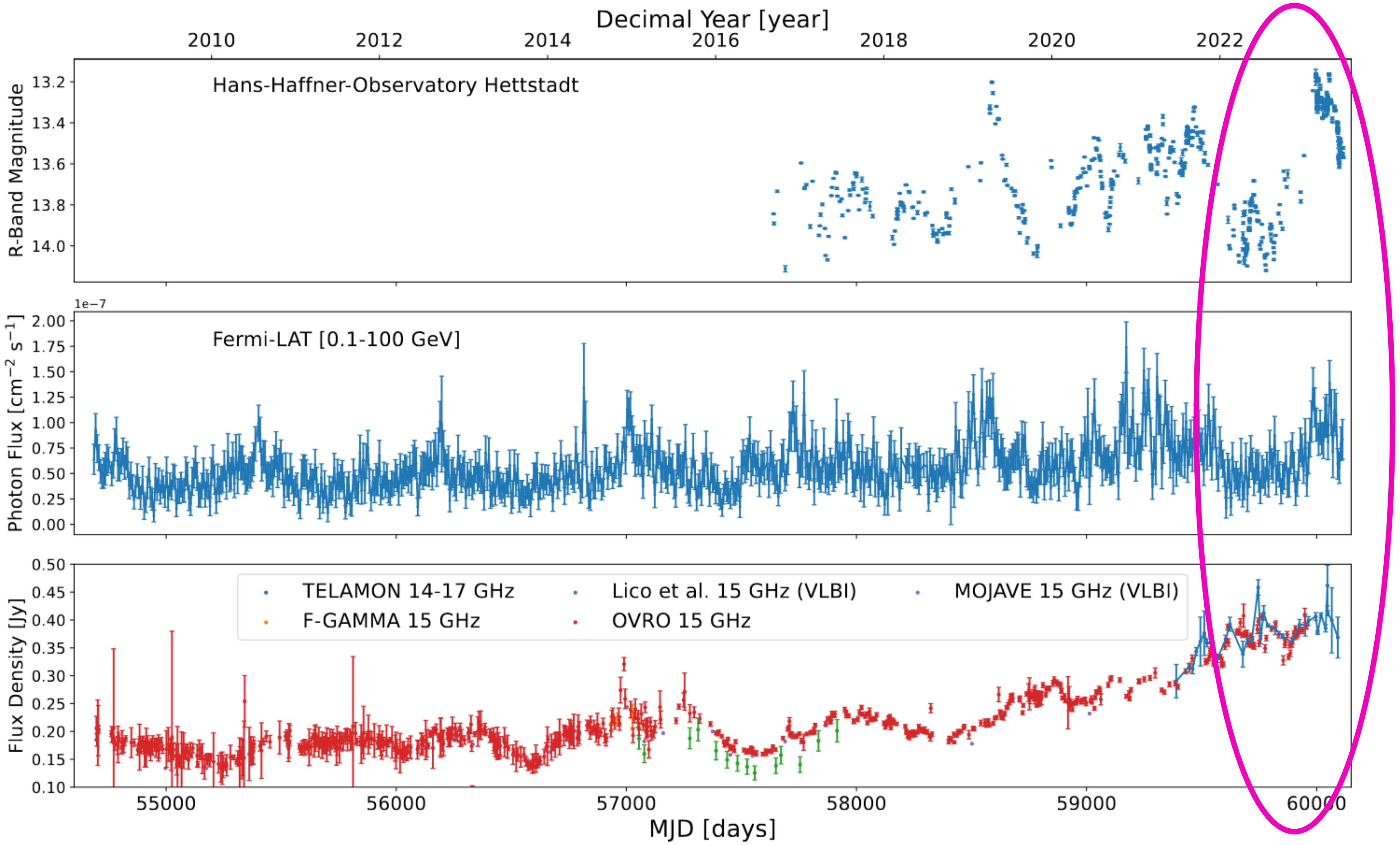

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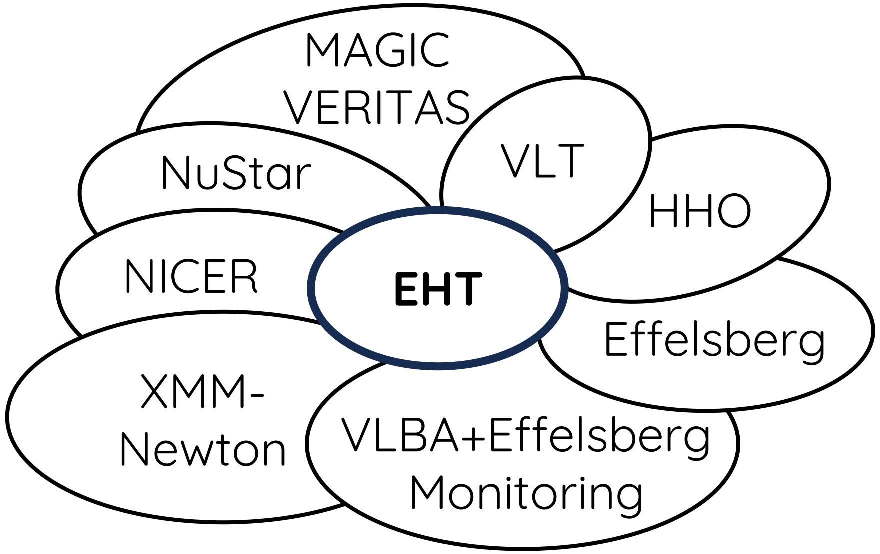

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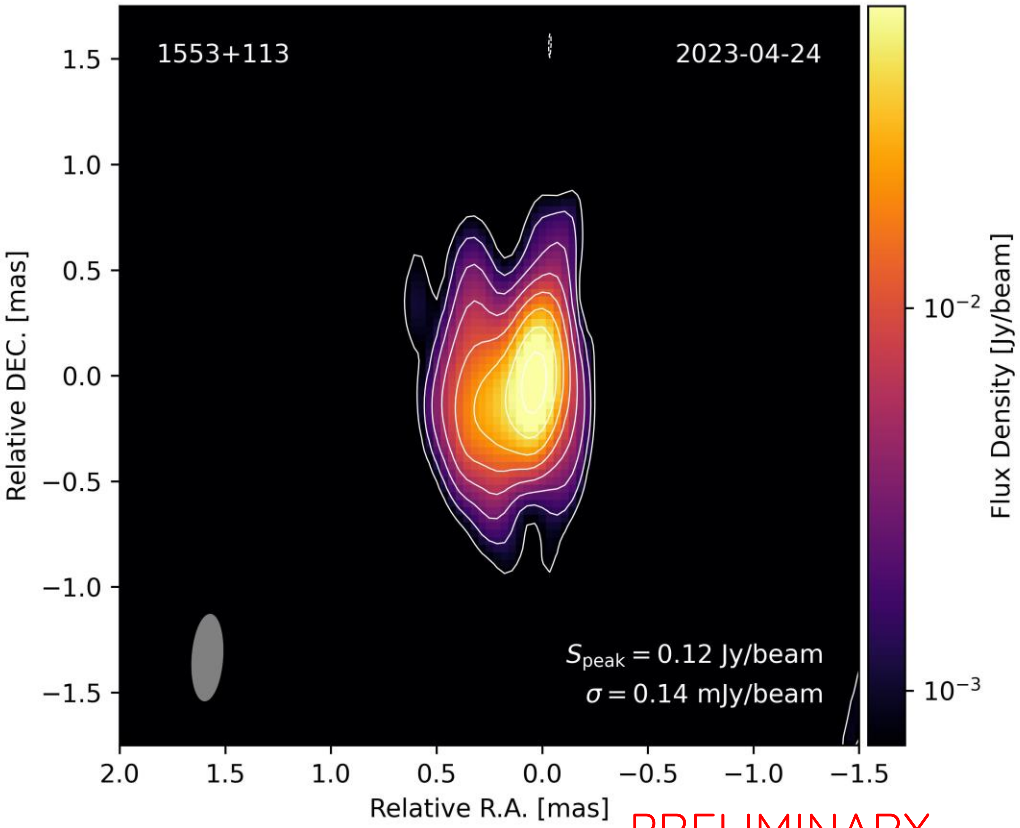
VLBA+Effelsberg 43 GHz



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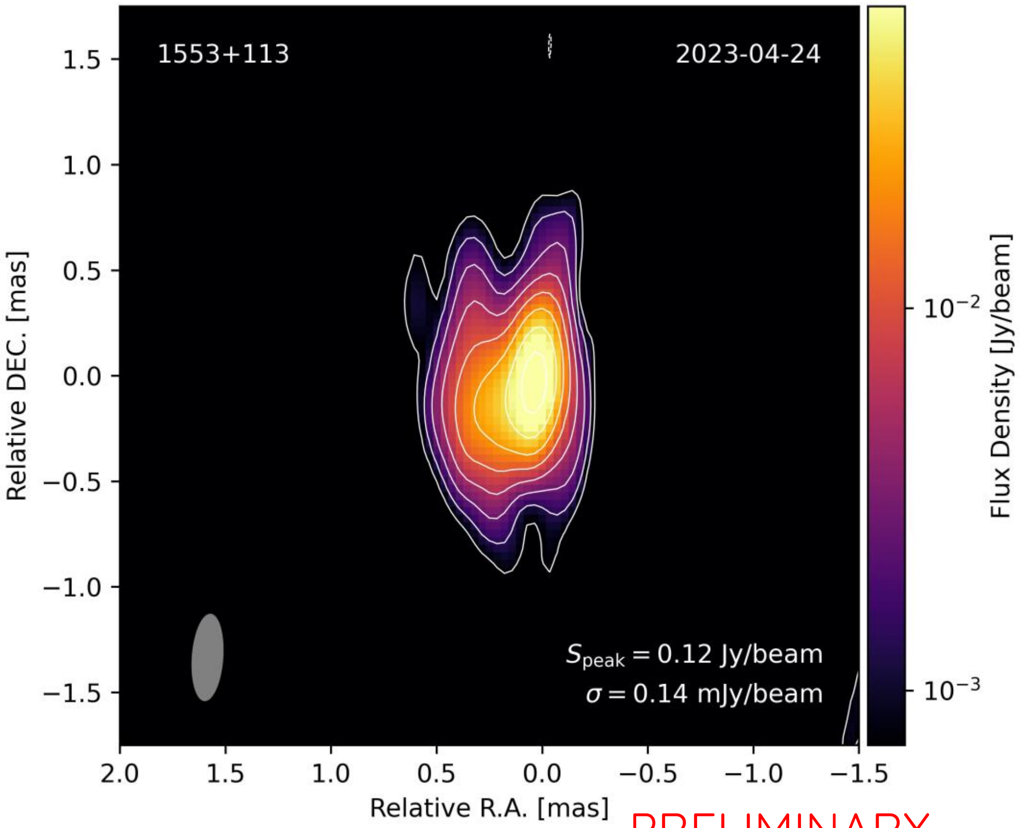

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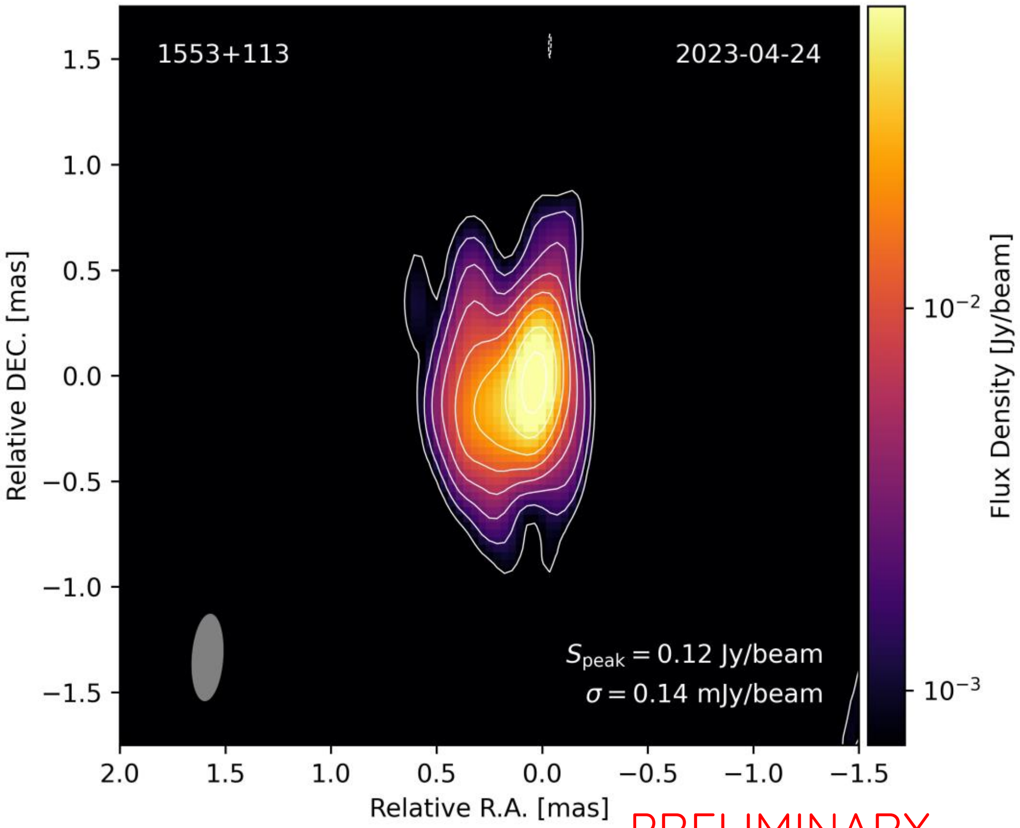

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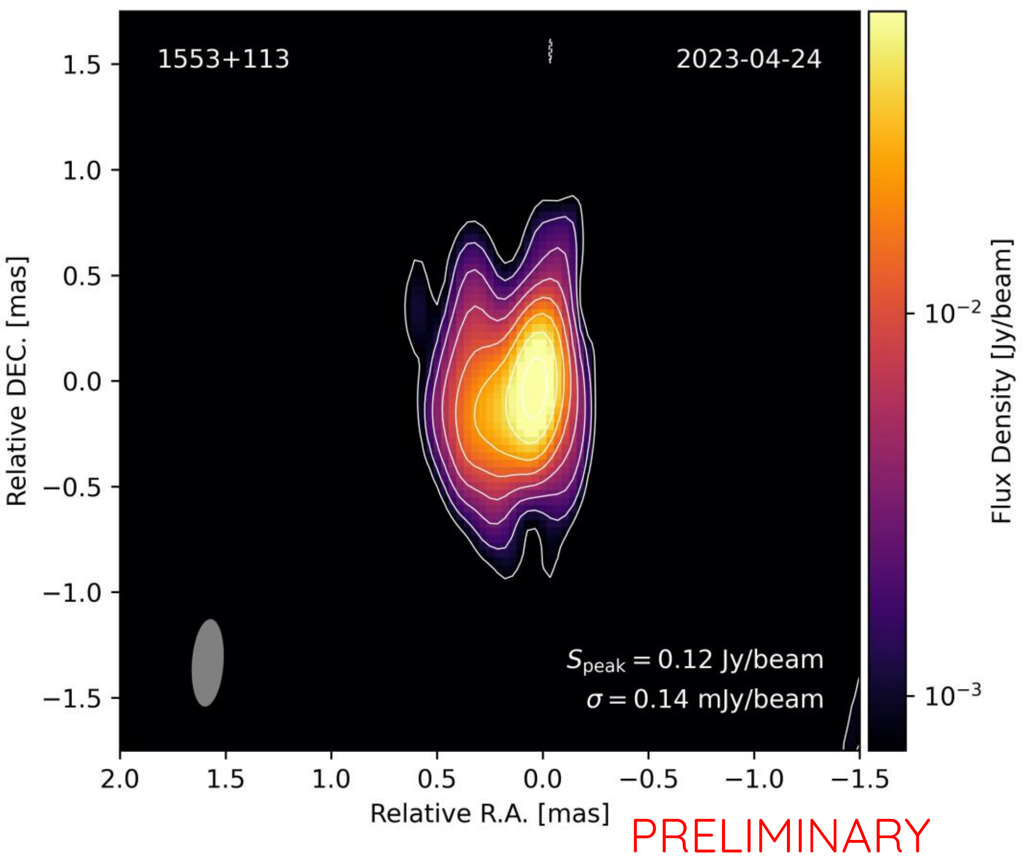
PG 1553+113: HBL and SMBBH

VLBA+Effelsberg 43 GHz



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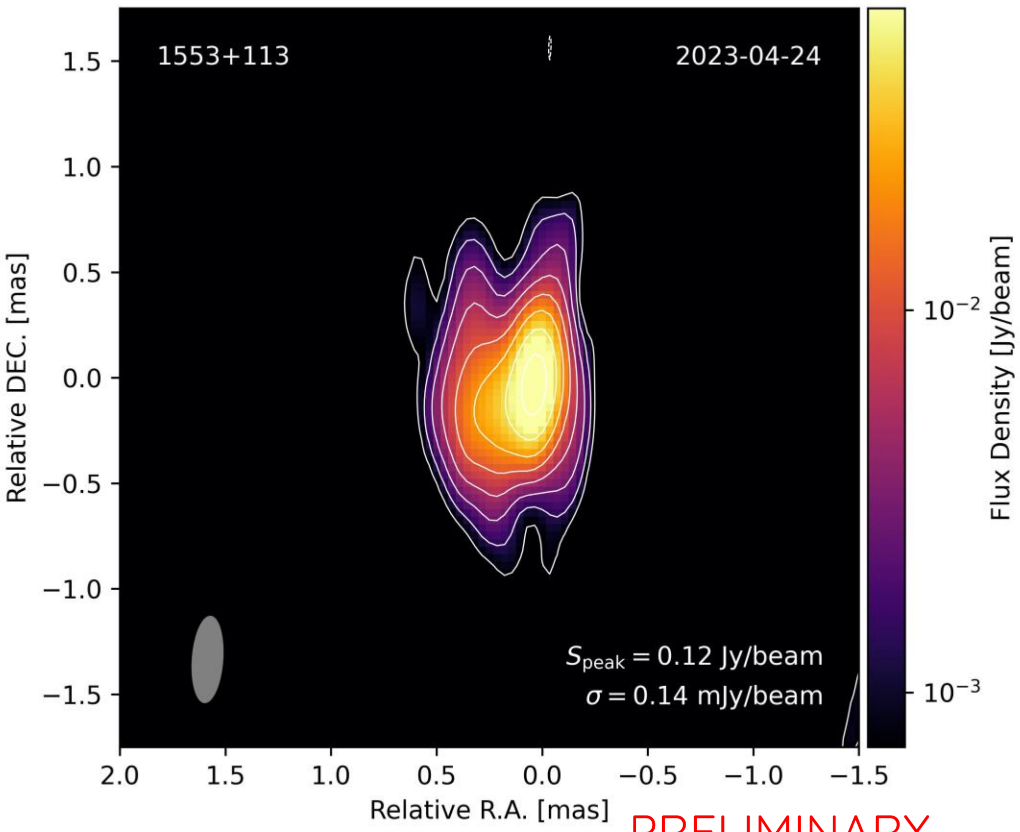

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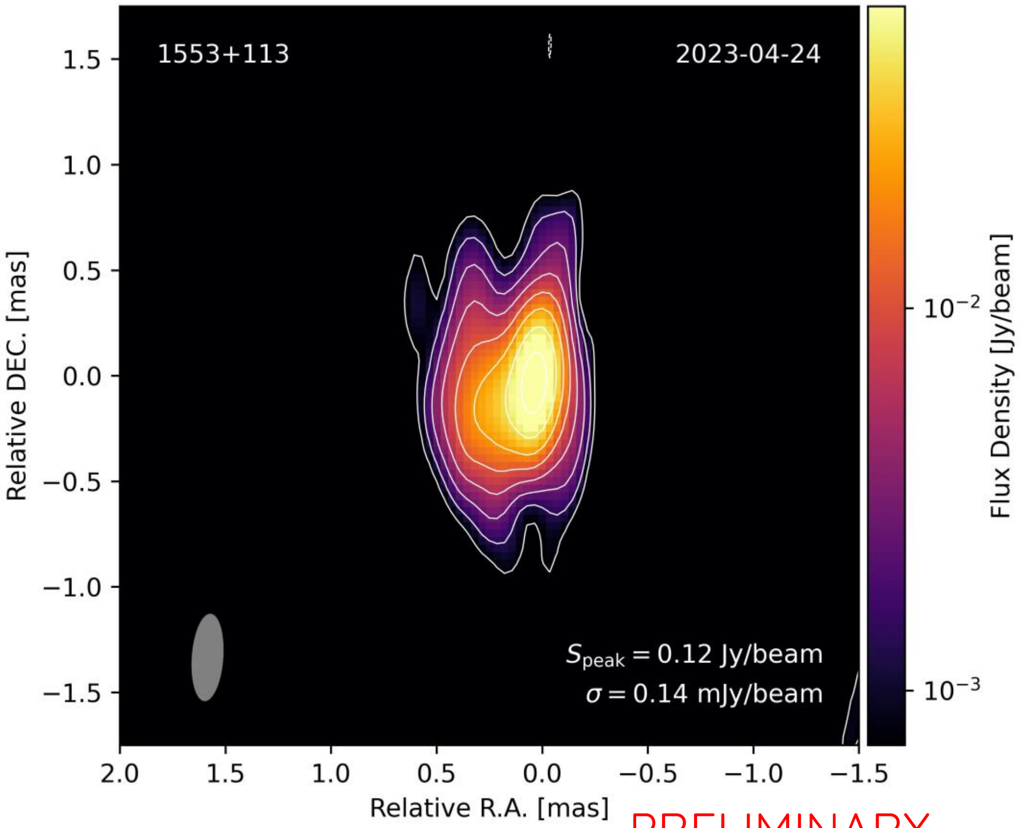

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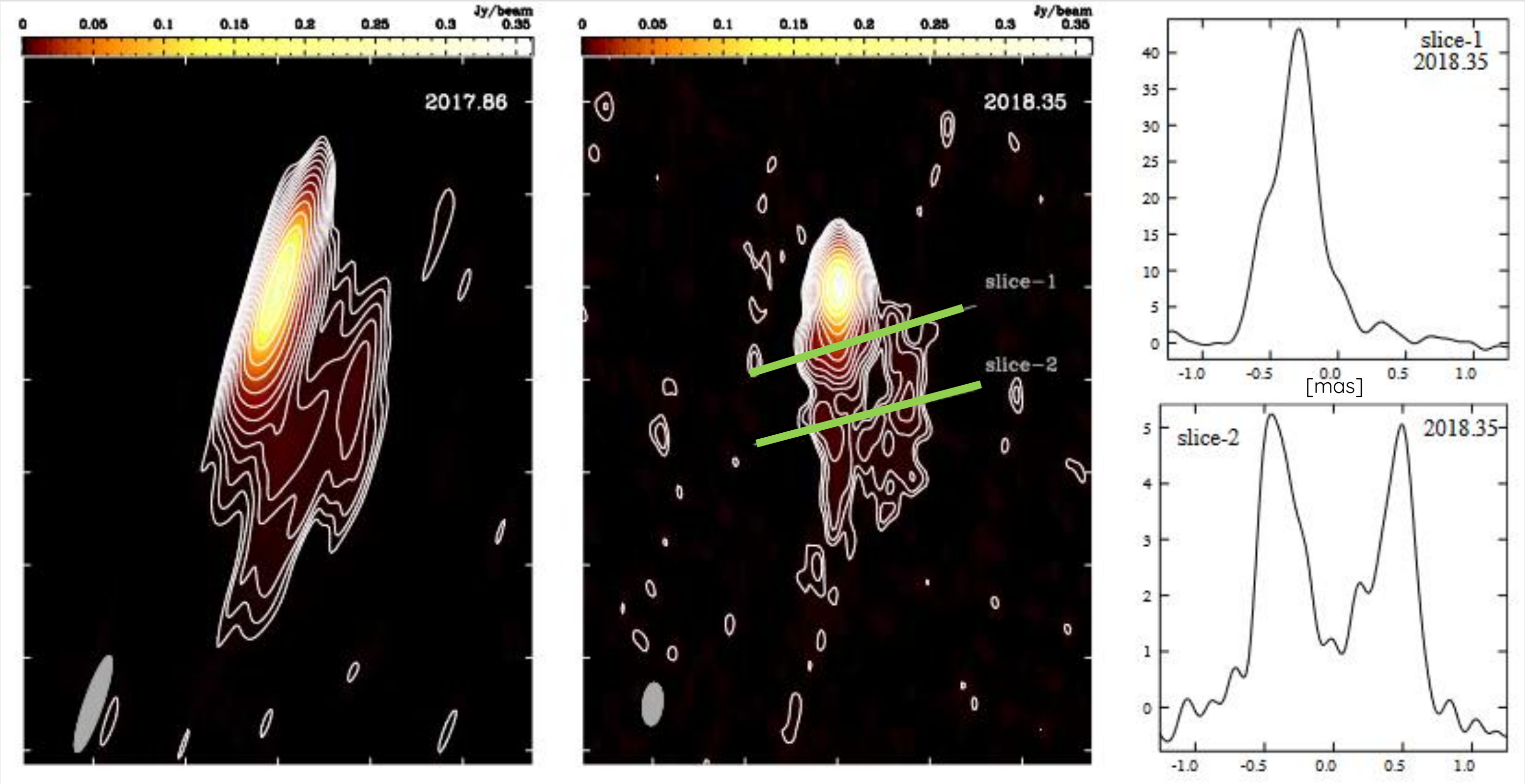
TXS 0506+056 (IC170922A)

Multiwavelength-Flare associated with Neutrino Event **IC170922A**



43 GHz VLBA images and surface brightness profiles of TXS 0506+056 after the associated neutrino event providing evidence of limb-brightening (Ros et al. 2020)

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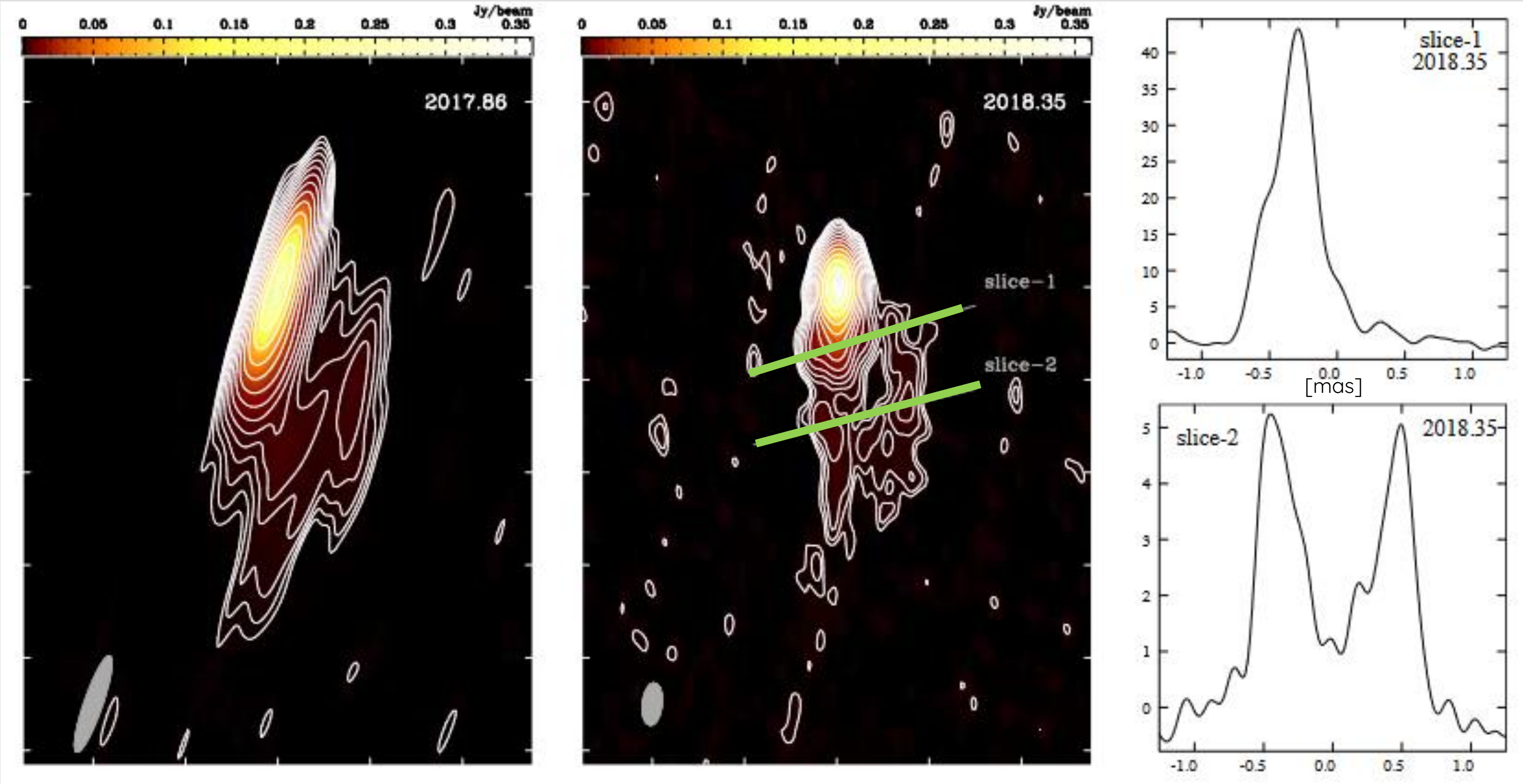

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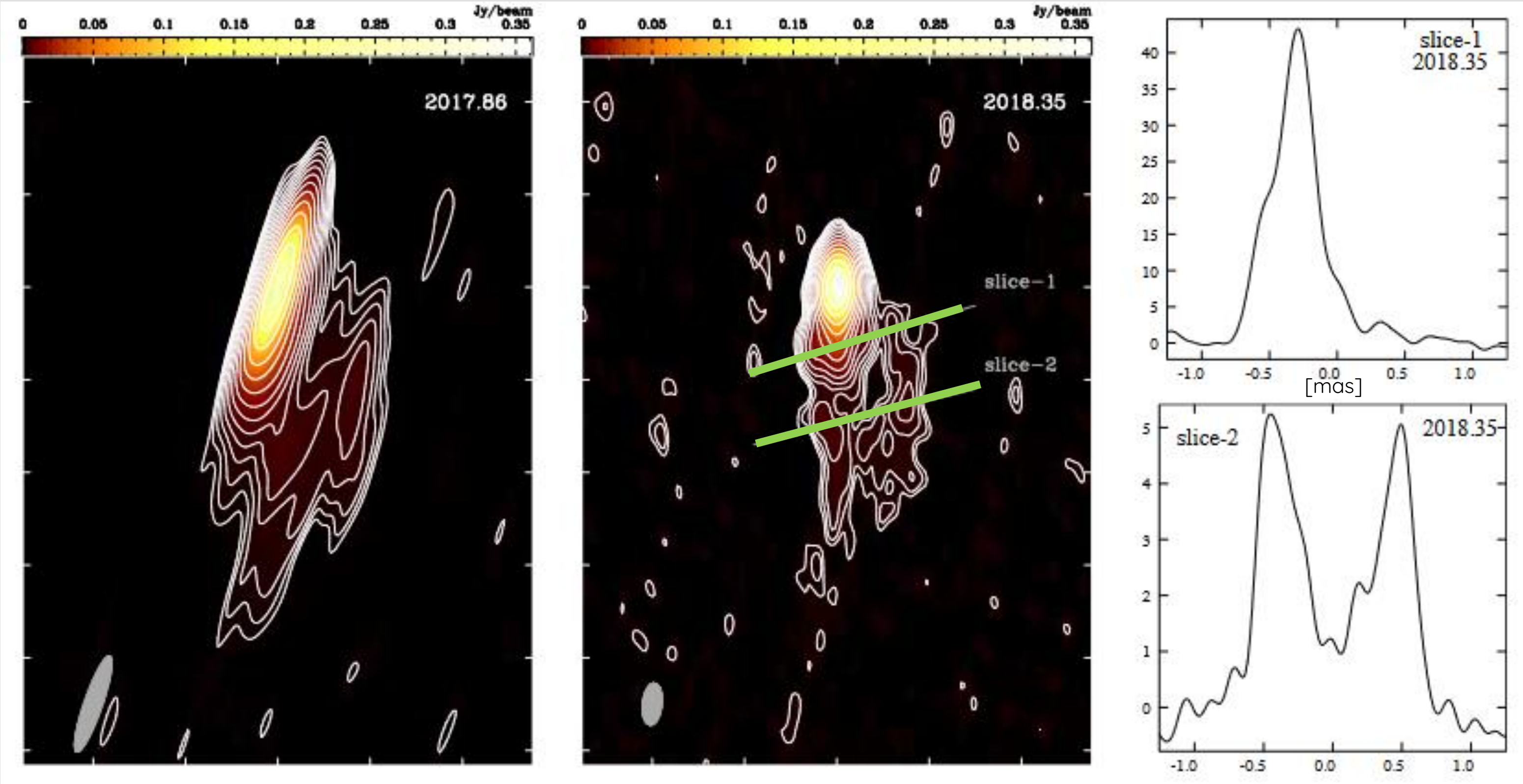

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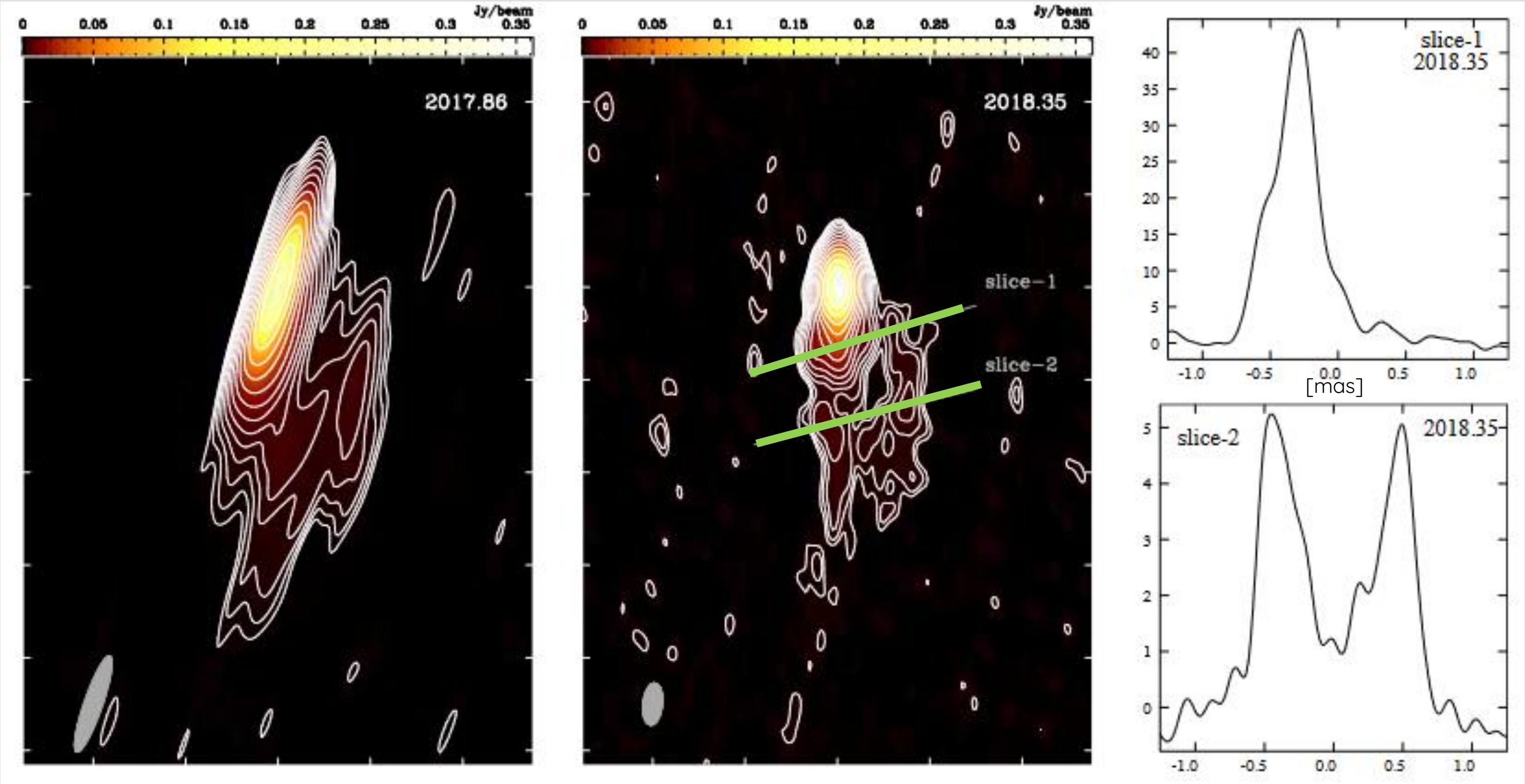

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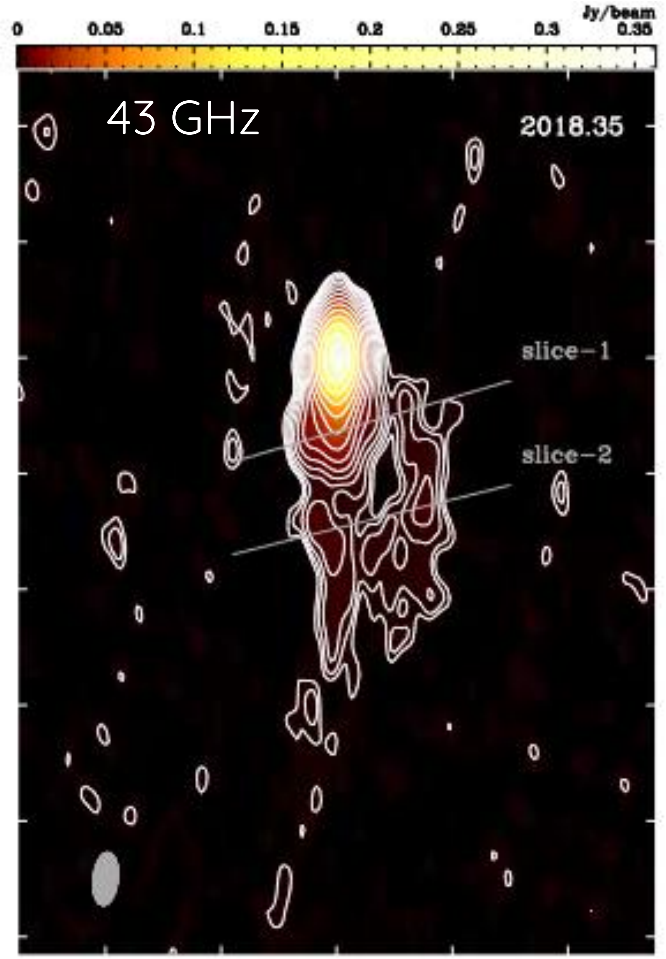
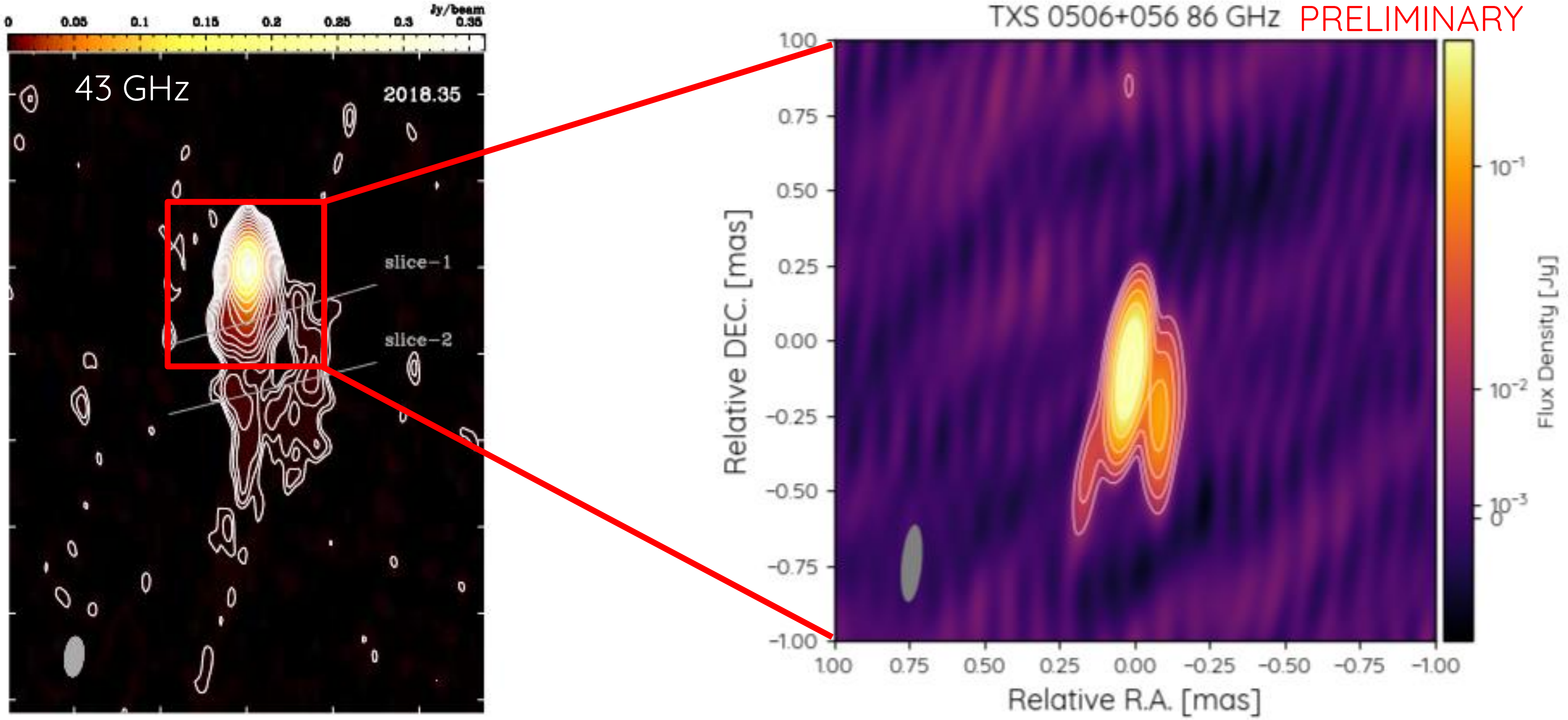
TXS 0506+056 at 86 GHz



TXS 0506+056 at 86 GHz



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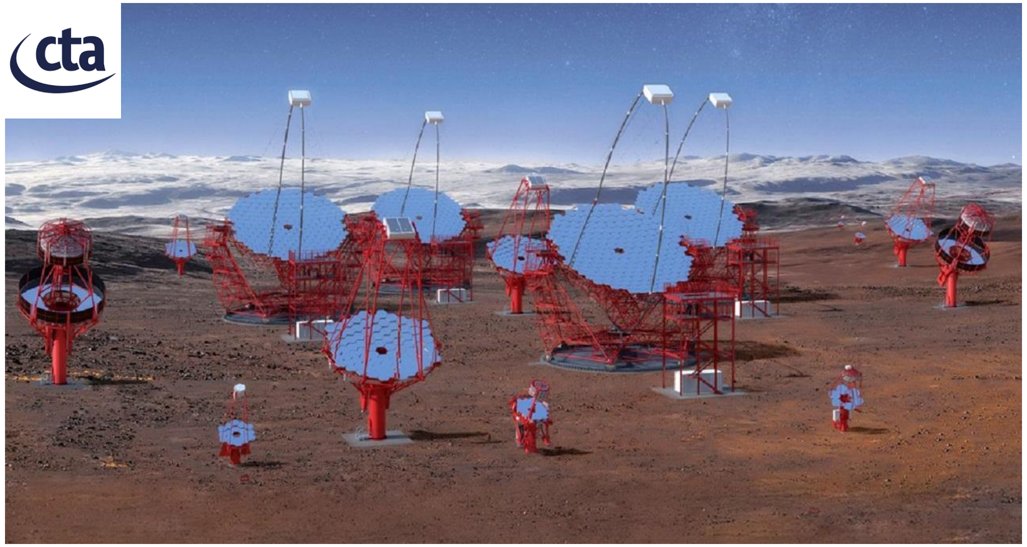

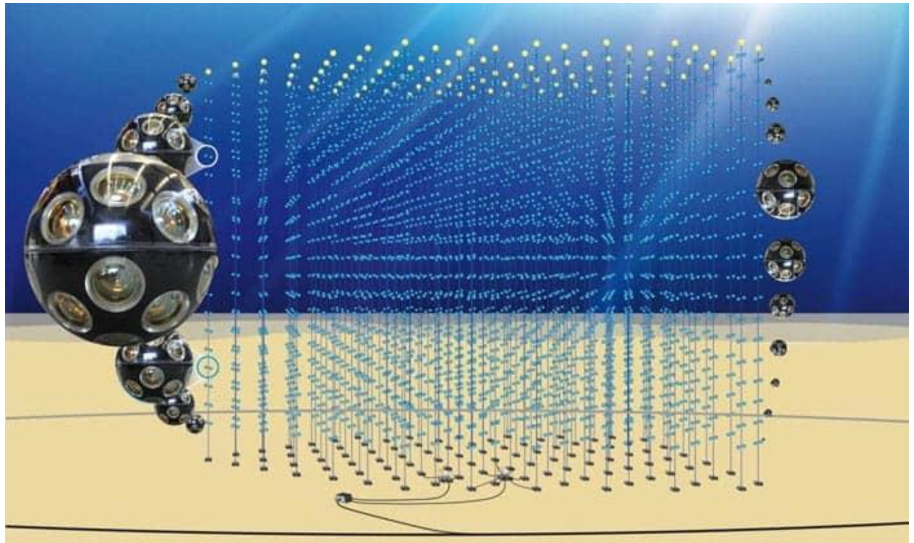

Thank you!

Questions?

Brightness Temperature Formulae

ASTRO WÜRZBURG

VLBI Brightness Temperature

$$T_b = \frac{2\ln 2}{\pi k}\frac{S_{\text{core}}\lambda^2(1+z)}{\theta_{\text{maj}}\theta_{\text{min}}}$$

Kovalev et al. (2005)

$$\delta_{VLBI} = (1+z)\frac{T_b}{T_{int}}$$

Variability Brightness Temperature

$$T_{\text{var}} = 1.47 \cdot 10^{13}\frac{D_L^2 \Delta S_{\text{ob}}(\nu)}{\nu^2 t_{\text{var}}^2 (1+z)^4}$$

Liodakis et al. (2017)

$$\delta_{\text{var}} = (1+z)\sqrt[3]{\frac{T_{\text{var}}}{T_{\text{eq}}}}.$$

VLBI Kinematics: Doppler Factor (Critical Angle)

$$\delta_{\text{crit}} = \sqrt{1+\beta_{\text{app}}^2}.$$

Fromm et al. (2013)