

# Gravitational Wave Astronomy

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April 9th, 2024 – High-energy astrophysics in the multi-messenger era

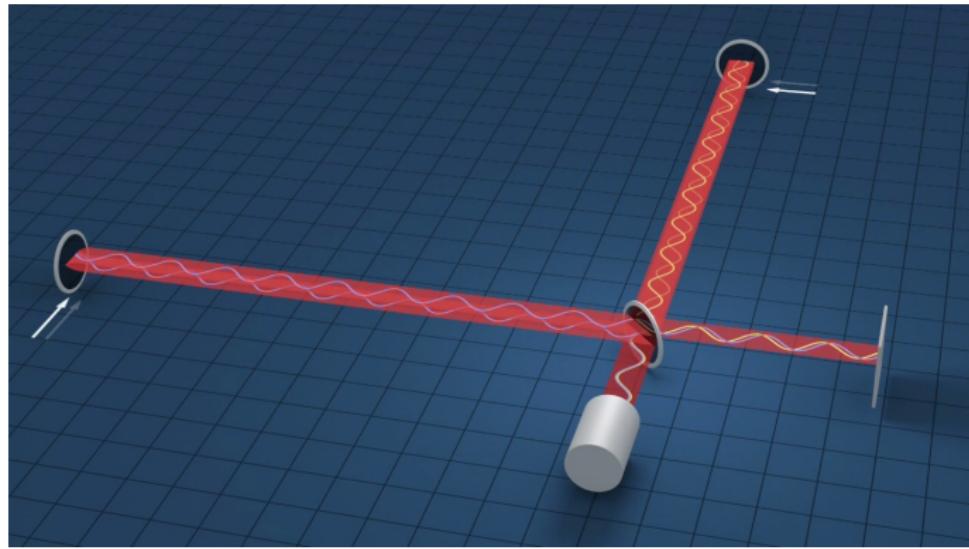
# A guide to GW observations

- 1 Experiment
- 2 Observations (astro perspective)
- 3 What we actually get from data
- 4 Field theory methods for modeling binary systems
- 5 Cosmology
- 6 Summary

# Outline

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# LIGO and Virgo (+KAGRA): very precise rulers



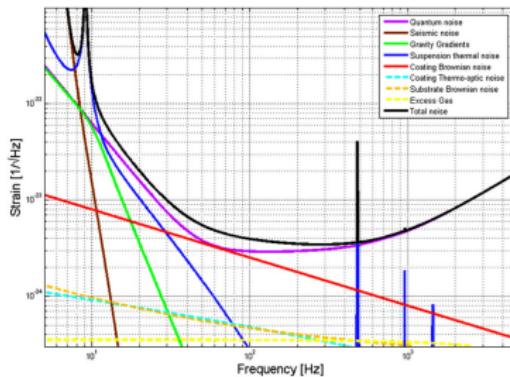
Robert Hurt (Caltech)

Light intensity  $\propto$  light travel difference in perpendicular arms

Effective optical path increased by factor  $N \sim 500$  via Fabry-Perot cavities

Phase shift  $\Delta\phi \sim 10^{-8}$  can be measured  $\sim 2\pi N \Delta L / \lambda \rightarrow \Delta L \sim 10^{-15} / N$  m

# Noise budget



Barriga et al., CQG (2013) 084005

Why  $\text{Hz}^{-1/2}$ ? Detector's FoM is *noise spectral density*  $S_n(f)$ :

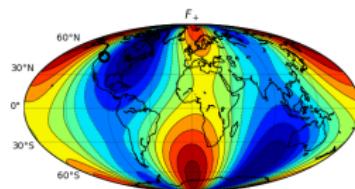
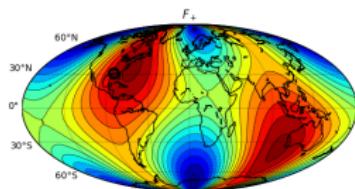
$$\langle \tilde{n}(f) \tilde{n}(f') \rangle = S_n(f) \delta(f - f')$$

i.e.  $S_n(f_i) \sim |\tilde{n}(f_i)|^2 \Delta f$ . Best sensitivity for an interferometer for  $\frac{\lambda_{GW}}{2} \gtrsim L \implies f_{GW\ best} \lesssim \frac{c}{4\pi L} \sim 160\text{Hz} \left(\frac{L}{150\text{km}}\right)^{-1}$

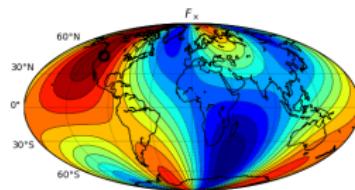
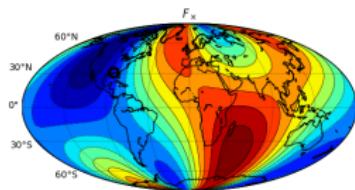
# Almost omnidirectional detectors

Detectors measure  $h_{det}$ : linear combination  $F_+ h_+ + F_x h_x$

$$\begin{matrix} -1 & 0 & 1 \\ F_+ \end{matrix}$$



$$F_x$$



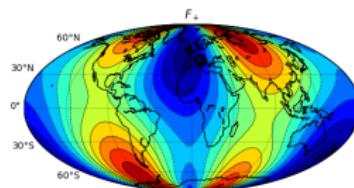
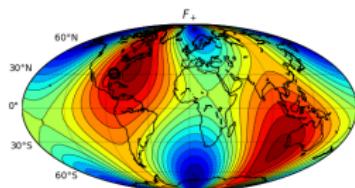
$h_{+,x}$  depend on source

pattern functions  $F_{+,x}$  depend on orientation source/detector

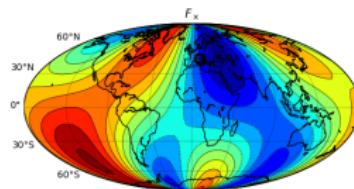
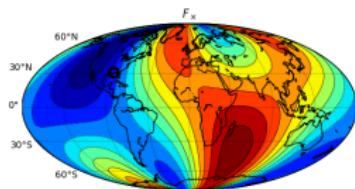
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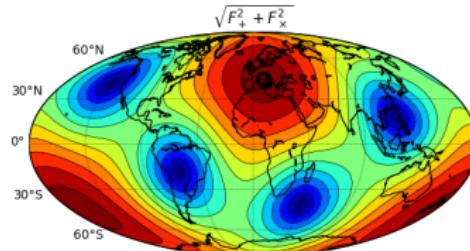
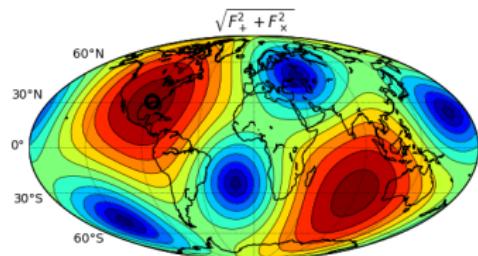
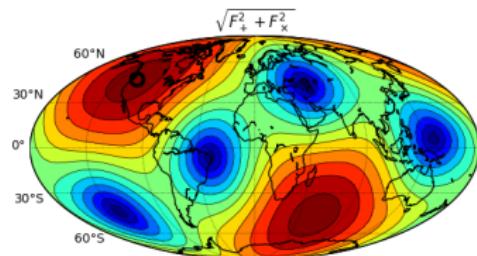
$$F_x$$



$h_{+,x}$  depend on source

pattern functions  $F_{+,x}$  depend on orientation source/detector

# Pattern functions: $\sqrt{F_+^2 + F_x^2}$



# The LIGO and Virgo observatories



- Observation run **O1** Sept '15 - Jan '16  
~ 130 days, with 49.6 days of actual data, PRX (2016) 4, 041014, **2 detectors, 3BBH**
- **O2** Dec. '16 – Jul'17 **2 det's + Aug '17 3 det's**  
**3(+4) BBH + 1BNS in double (triple) coinc.**
- **O3a:** **3 detectors, Apr - Sep 2019, 39 detections**
- **O3b:** Nov 1st – Mar 27th 2020 → 90 detections
- **O4a:** Ongoing (since May 24th) → end 2024

# KAGRA



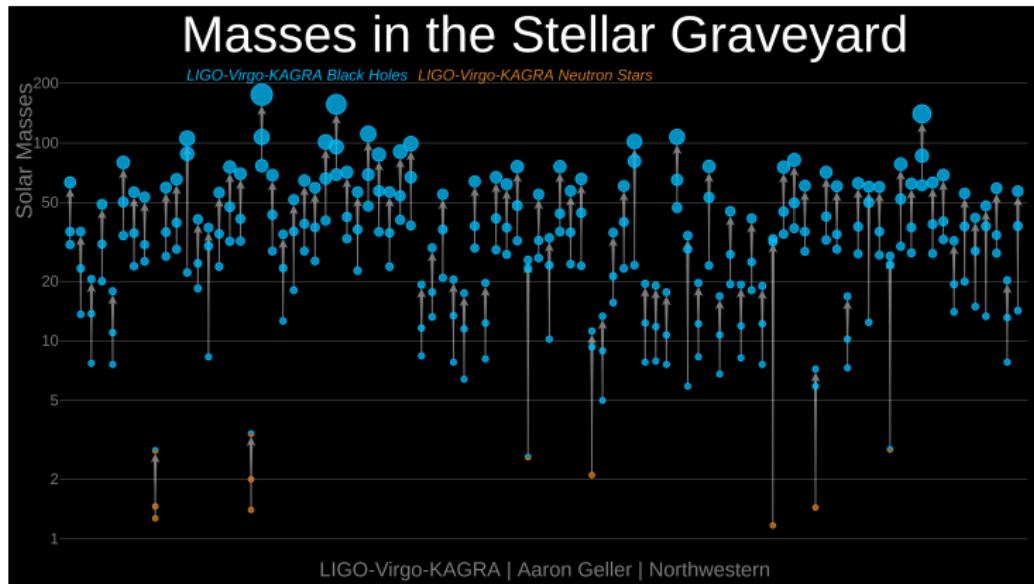
Additional *underground, cryogenic* detector

KAGRA Collaboration, Galaxies 10 (2022) 3, 63

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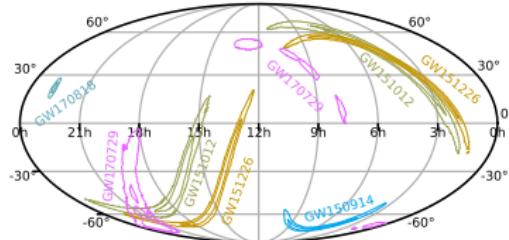
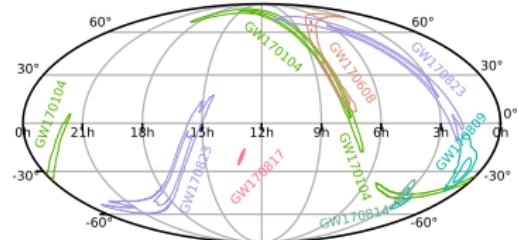
# Stellar ( $< 100M_{\odot}$ ) compact object with known masses



Frequency  $10\text{-}10^3$  Hz determines size of sources

Remnant of various GW events represent first **Intermediate Mass Black Holes** ( $> 10^2 M_{\odot}$ ) – SuperMassive BHs  $\gtrsim 10^5 M_{\odot}$  (up to  $10^9 M_{\odot}$ )

# Sky localization: sample events



One needs 3 detectors to triangulate the source (useful info from pattern functions)

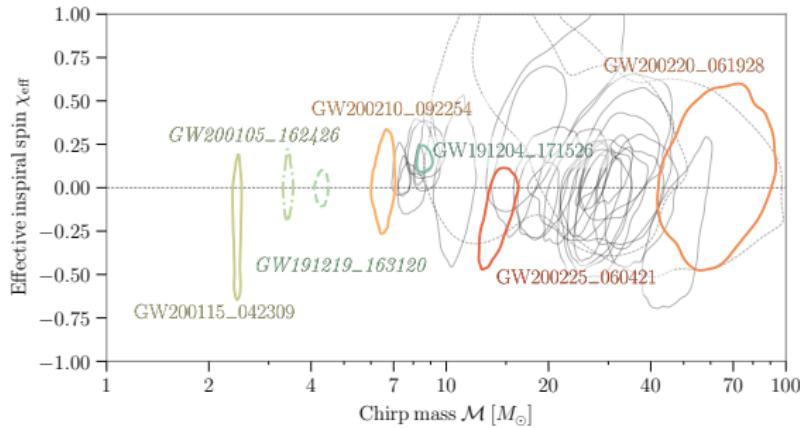
Distances between 40 Mpc and  $\sim 5$  Gpc ( $\pm 20\%$ )

(Milky Way's size  $\sim 30$ kpc, Galaxy-Galaxy  $\sim 4$ Mpc)

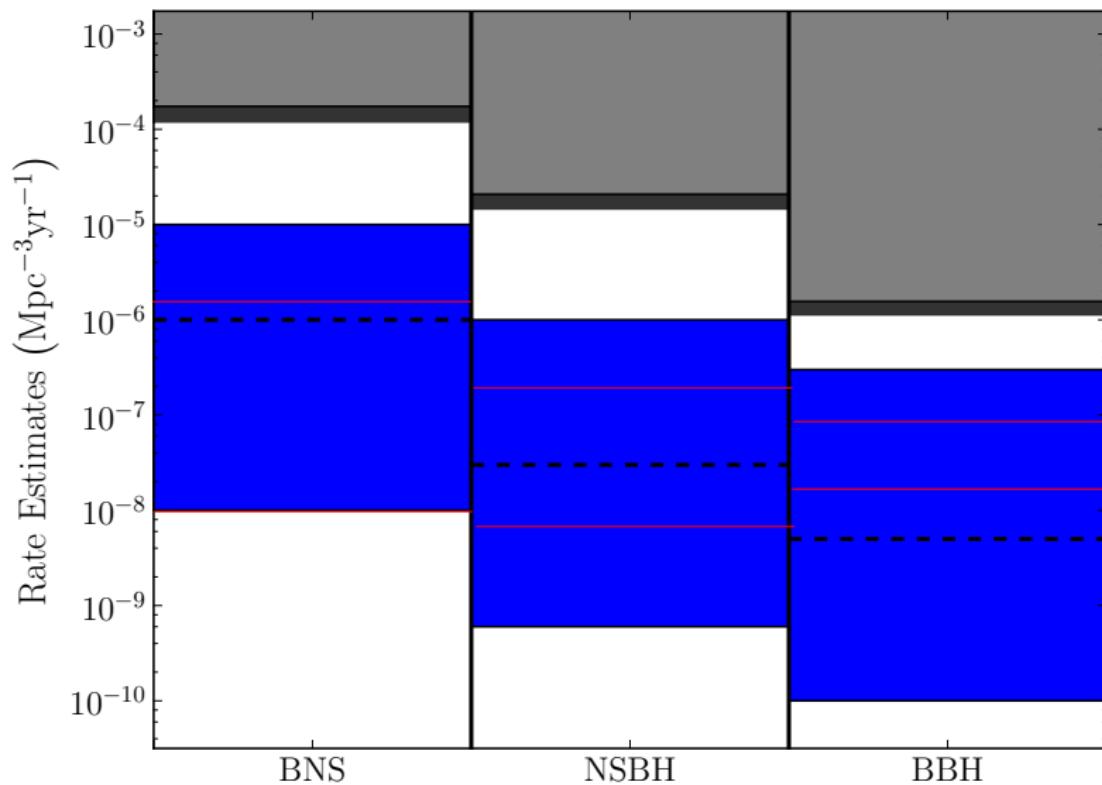
Image by Leo Singer, <http://www.ligo.org>

# Little is known so far about spins

$$\vec{S}_i = m_i^2 \chi_i, \chi_{\text{eff}} \equiv \frac{m_1 \vec{\chi}_1 \cdot \vec{L} + m_2 \vec{\chi}_2 \cdot \vec{L}}{M}, M = m_1 + m_2$$



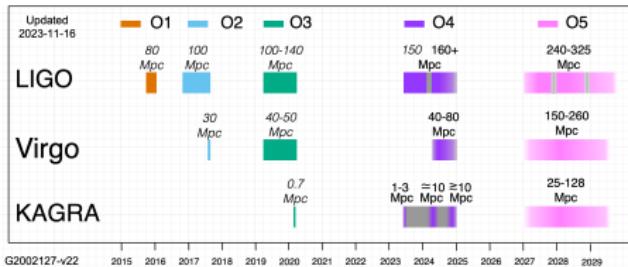
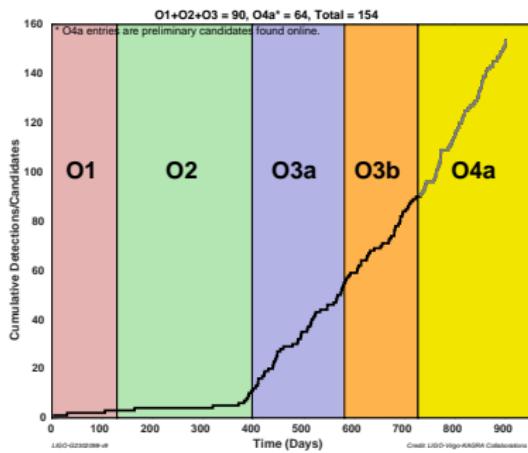
More unequal-masses systems bear larger spin imprint:  $|\vec{L}| \sim \frac{m_1 m_2}{\nu}$



Astro predictions, measures from O1/O2/O3. Galaxy density  $\sim 2 \times 10^{-2} \text{ Mpc}^{-3}$

LIGO/Virgo CQG ('10) 27 173001, PRX ('16), APJ (2016), PRL 119 ('17)

# LIGO/Virgo/KAGRA's prospects



<https://dcc.ligo.org/LIGO-G2302098>

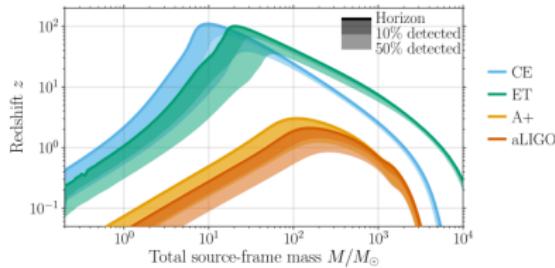
# Future with ET and LISA looks very loud

Future 3rd generation detectors (Einstein Telescope, Cosmic Explorer)/space telescope LISA will detect CBC signals with SNR  $10 - 10^2$ , with few golden events with SNR  $\sim 10^3$ .

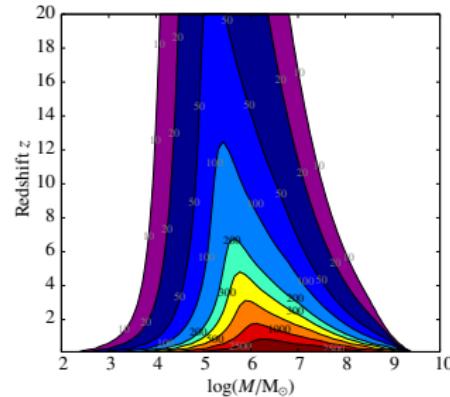
Templates few % accurate OK for characterising a source with SNR  $O(10)$  (typical for LIGO/Virgo)

for SNR  $\sim 10^3$  residual after extracting that source will have SNR  $\sim O(10)$

- ➊ biasing parameter estimation
- ➋ contaminating the extraction of additional sources.



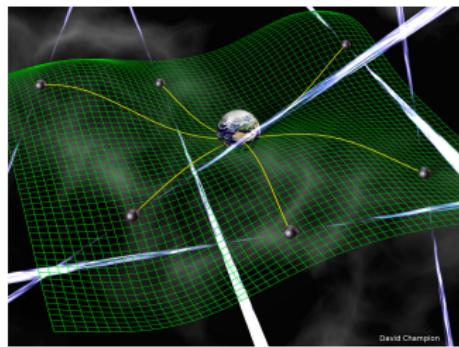
Hall, Evans CQG (2019), 1902.09485



Amaro-Seoane+, GW Notes 6 (2013),

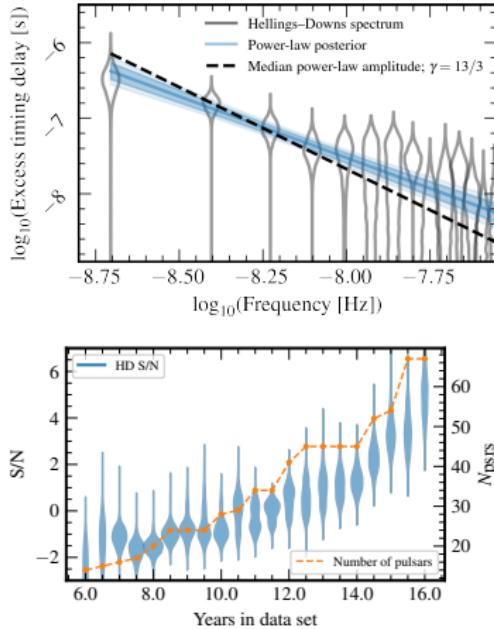
# Nanograv (& PTA)

Monitoring irregularities in pulsar signals one can infer GW strength:



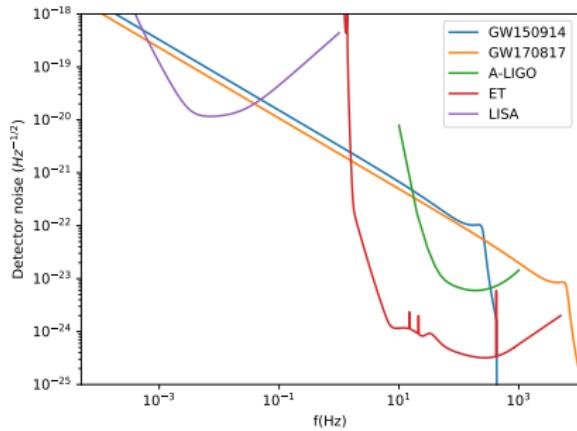
Credit: David Champion

$$T = 16.03 \text{ yr} = \frac{1}{2\text{nHz}}, f_i = i/T$$



Nanograv, APJ Lett. (2023) 2306 16213 ↗

# Sensitivities and duration



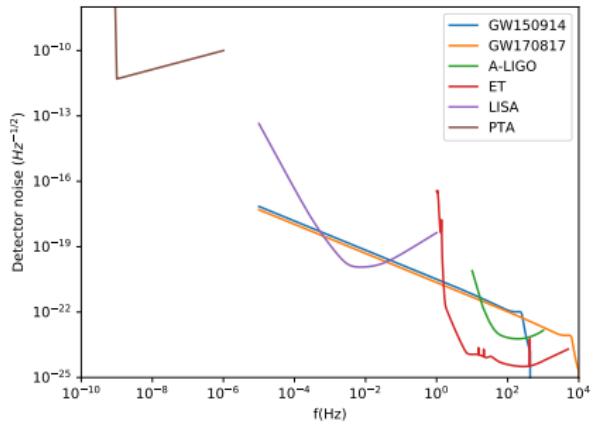
$$\tilde{h}(f) \sim \frac{f^{-7/6} (GM_c)^{5/6}}{D_L} e^{i\psi(f)}$$

Signal duration controlled by  $\frac{5}{256\pi} (\pi M_c f_i)^{-5/3} = 30 \times \frac{1}{\eta} \left( \frac{M}{20M_\odot} \right)^{-5/3} \left( \frac{f_i}{20\text{Hz}} \right)^{-5/3}$

$$\Delta t_{i \rightarrow f} \sim \frac{5}{256\pi} (\pi GM_c)^{-5/3} \left( f_i^{-8/3} - f_m^{-8/3} \right) \rightarrow$$

$$\frac{5}{256\pi} (\pi M_c f_i)^{-5/3} \times \begin{cases} \frac{1}{f_i} & \Delta t < t_{exp} \\ \frac{\Delta f}{f_i^2} & \Delta t > t_{exp} \end{cases}$$

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# Wave generation: localized sources

Einstein formula relates  $h_{ij}$  to the source quadrupole moment  $Q_{ij}$

$$\begin{aligned} Q_{ij} &= \int d^3x \rho \left( x_i x_j - \frac{1}{3} \delta_{ij} x^2 \right), \quad v^2 \simeq G_N M / r, \quad \eta \equiv m_1 m_2 / M^2 \\ h_{ij} &\sim g(\theta_{LN}) \frac{2G_N}{D} \frac{d^2 Q_{ij}}{dt^2} \simeq \frac{2G_N \eta M v^2}{D} \cos(2\phi(t)) \\ f &= 2\text{kHz} \left( \frac{r}{30\text{Km}} \right)^{-3/2} \left( \frac{M}{3M_\odot} \right)^{1/2} < f_{Max} \simeq 12\text{kHz} \left( \frac{M}{3M_\odot} \right)^{-1} \\ v &= 0.3 \left( \frac{f}{1\text{kHz}} \right)^{1/3} \left( \frac{M}{M_\odot} \right)^{1/3} < \frac{1}{\sqrt{6}} \end{aligned}$$

Geometric factor  $g(\theta_{LN})$  takes account of **transversality** projection  
(angular momentum  $L$  of the binary, observation direction  $N$ )

$$\begin{aligned} h_+ &\sim \frac{1 + \cos^2(\theta_{LN})}{2} \eta \frac{Mv^2}{D} \cos \phi(t_s/M, \eta, S_i^2/m_i^4, \dots) \\ h_\times &\sim \cos(\theta_{LN}) \eta \frac{Mv^2}{D} \sin \phi(t_s/M, \eta, \dots) \end{aligned}$$

Amplitudes of 2 polarizations modulated by  $\theta_{LN}$  ( $h_+$   $\nearrow$  for  $\theta_{LN} \searrow 0$ ), never both vanishing  
unlike dipolar motion for the electromagnetic case  $\Lambda$  breaks scaling  $Mvs.1+z$   
degeneracy.

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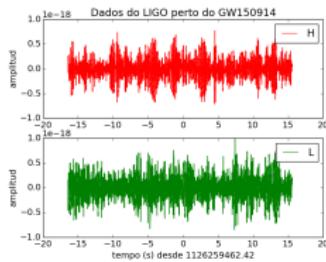
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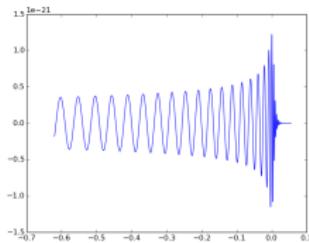
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$h$  sensitive to **red-shifted** masses  $M \rightarrow M(1+z) \equiv \mathcal{M}$

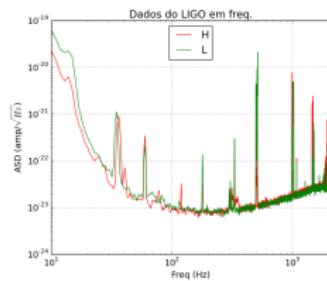
# Matched filtering



X

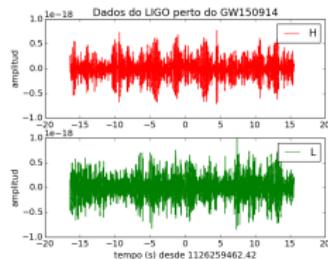
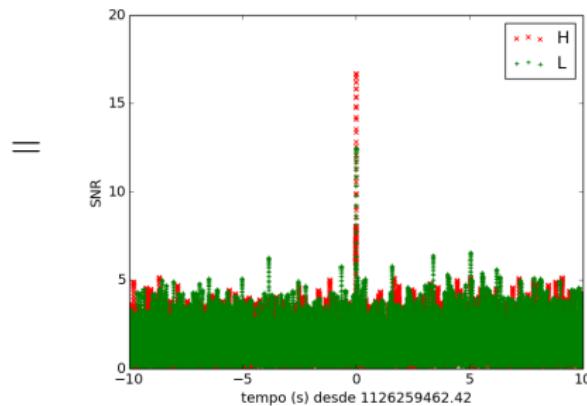
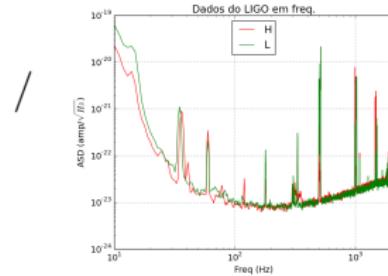
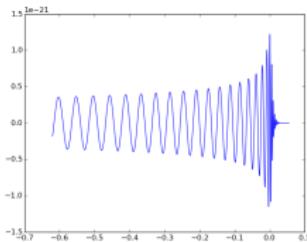


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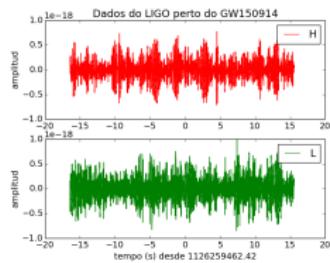
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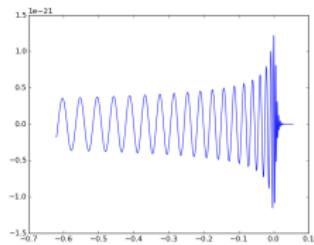
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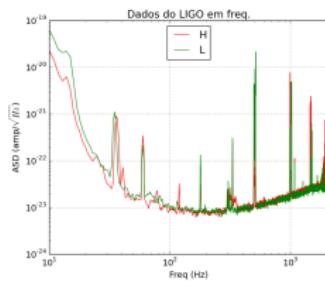
# The importance of theoretical modeling



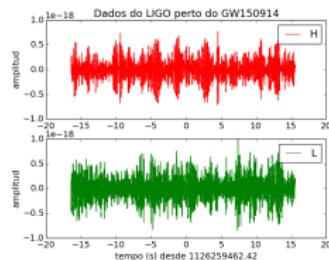
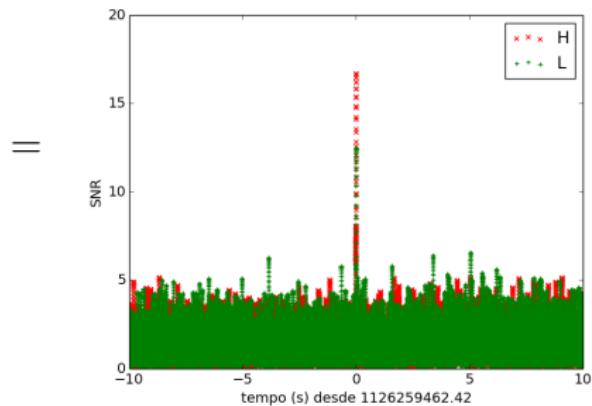
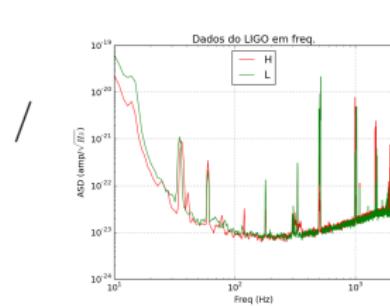
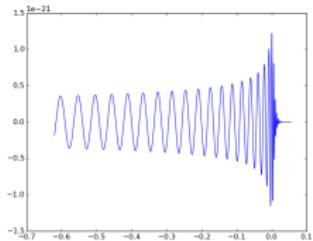
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# The importance of theoretical modeling

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Data from <https://losc.ligo.org/events/GW150914/>

# Fundamental GR: inspiral analytic model

Inspiral  $h = A \cos(\phi(t))$        $\frac{\dot{A}}{A} \ll \dot{\phi}$

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \quad \eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$\begin{aligned} E(v) &= -\frac{1}{2}\eta M v^2 (1 + \#(\eta, S_i/m_i^2)v^2 + \#(\eta, S_i/m_i^2)v^4 + \dots) \\ P(v) \equiv -\frac{dE}{dt} &= \frac{32}{5G_N} v^{10} (1 + \#(\eta, S_i/m_i^2)v^2 + \#(\eta, S_i/m_i^2)v^3 + \dots) \end{aligned}$$

$E(v)$ ( $P(v)$ ) known up to 3(3.5)PN

$$\begin{aligned} \frac{1}{2\pi} \phi(T) &= \frac{1}{2\pi} \int^T \omega(t) dt = - \int^{v(T)} \frac{\omega(v) dE/dv}{P(v)} dv \\ &\sim \int (1 + \#(\eta, S_i/m_i^2)v^2 + \dots + \#(\eta, S_i/m_i^2)v^6 + \dots) \frac{dv}{v^6} \end{aligned}$$

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Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \quad \eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$\begin{aligned} E(v) &= -\frac{1}{2} \eta M v^2 (1 + \#(\eta, S_i/m_i^2)v^2 + \#(\eta, S_i/m_i^2)v^4 + \dots) \\ P(v) \equiv -\frac{dE}{dt} &= \frac{32}{5G_N} v^{10} (1 + \#(\eta, S_i/m_i^2)v^2 + \#(\eta, S_i/m_i^2)v^3 + \dots) \end{aligned}$$

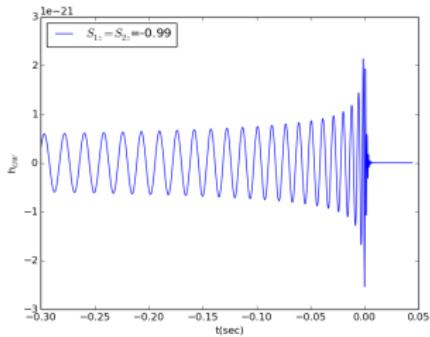
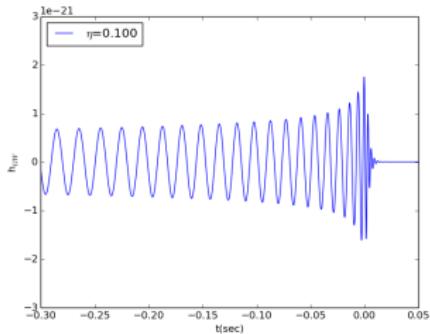
$E(v)(P(v))$  known up to 3(3.5)PN

$$\begin{aligned} \frac{1}{2\pi} \phi(T) &= \frac{1}{2\pi} \int^T \omega(t) dt = - \int^{v(T)} \frac{\omega(v) dE/dv}{P(v)} dv \\ &\sim \int (1 + \#(\eta, S_i/m_i^2)v^2 + \dots + \#(\eta, S_i/m_i^2)v^6 + \dots) \frac{dv}{v^6} \end{aligned}$$

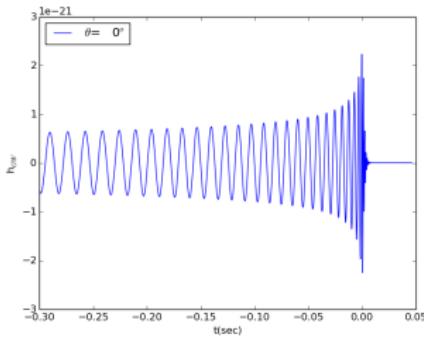
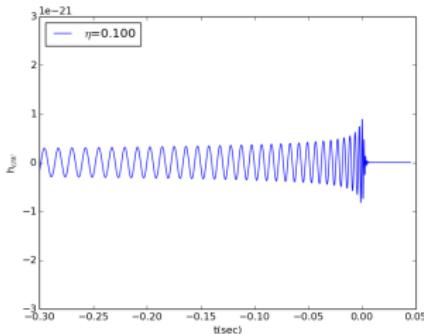
PN Coefficients (absorption  $\sim v^8$ , tidal  $\sim v^{10}$ )

# Looking for source fingerprints

$M_c$  fixed

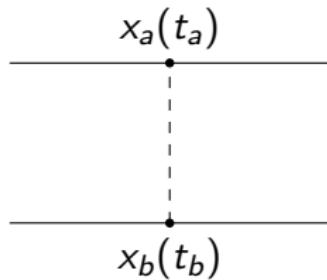


$M$  fixed



# 1PM potential

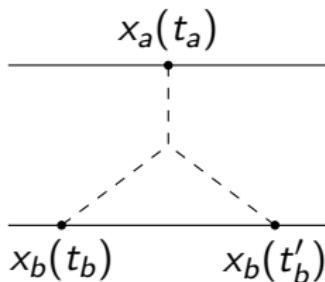
Out of different ways of computing 2-body Post-Minkowskian expansion  
e.g. 1PM  $O(G_N^1)$  potential gravity coupled to particle world-lines:



$$V_{PM}^{(1)}(x_a^\mu - x_b^\mu) = G_N T_{\mu\nu}^a T_{\rho\sigma}^b \Delta^{\mu\nu, \rho\sigma} \int \frac{d^4 k}{(2\pi)^4} \frac{e^{ik^\mu(x_{a\mu} - x_{b\mu})}}{|\mathbf{k}|^2 - k_0^2 + \epsilon \text{ terms}}$$

# PM complicates

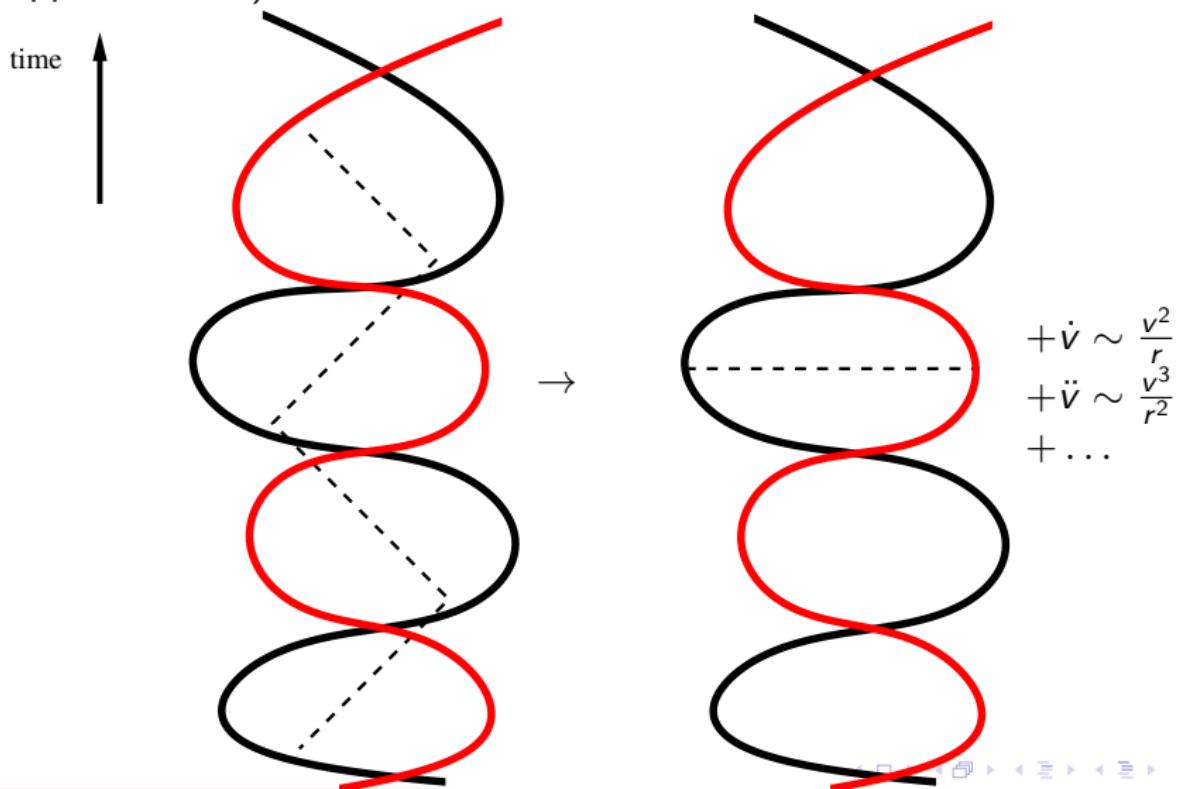
At higher order things rapidly complicate



$$\begin{aligned}
 V^{(2PM)} &\supset G_N^2 m_1 m_2^2 \int d^4 p e^{ip_\mu(x_a^\mu(t_a) - x_b^\mu(t_b))} \frac{p^\alpha p^\beta}{p^2} \int d^4 k \frac{e^{ik^\mu(x_b^\mu(t_b) - x_b^\mu(t'_b))}}{(p - k)^2 k^2} \\
 &= G_N^2 m_1 m_2^2 \int d^4 p e^{ip_\mu(x_a^\mu(t_a) - x_b^\mu(t_b))} \frac{p^\alpha p^\beta}{p^2} \Delta(p^\mu(x_{2\mu}(t_b) - x_{2\mu}(t'_b)))
 \end{aligned}$$

These kinds of “conservative” diagrams computed up to 4PM order

post-Newtonian approximation trades knowledge over the full trajectory with knowledge of all derivatives of the trajectory at equal time (PN approximation)



# Near zone conservative dynamics



The potential  $V$  (via Feynman Green function): \_\_\_\_\_

$$\begin{aligned}
 V &\propto \int dk_0 d^3k \frac{e^{-ik_0 t_{12} + i\vec{k} \cdot (\vec{x}_1(t_1) - \vec{x}_2(t_2))}}{k^2 - k_0^2 - i\epsilon} = \int dk_0 d^3k \frac{e^{-ik_0 t_{12} + i\vec{k} \cdot \vec{x}_{12}}}{k^2} \left(1 + \frac{k_0^2}{k^2} + \dots\right) \\
 &= \delta(t_1 - t_2) \int d^3k \frac{e^{i\vec{k} \cdot \vec{x}_{12}}}{k^2} \left(1 + \frac{\partial_{t_1} \partial_{t_2}}{k^2} + \dots\right) \\
 &= \int d^3k \frac{e^{i\vec{k} \cdot \vec{x}_{12}}}{k^2} \left(1 - \frac{\vec{k} \cdot \vec{v}_1 \vec{k} \cdot \vec{v}_2}{k^2} + \dots + \frac{\vec{k} \cdot \frac{d^{n-1} \vec{v}_1}{dt^{n-1}} \vec{k} \cdot \frac{d^{n-1} \vec{v}_2}{dt^{n-1}}}{k^{2n}}\right)
 \end{aligned}$$

"Breaking" the propagator enormous simplification, but introduces **spurious divergences**:  
Near zone amplitude integrands clearly bad behaved for  $k \rightarrow 0$  at high PN-order  
Straightforward fix: add the contribution of far-zone, for demonstration see e.g.

Manohar+ '07, Jentzen '12, Blumlein+ '20

# EFT and amplitude: tale of a happy marriage

The **main** obstruction to scalability of the NRGR PN calculation program is the computation of **master integrals**

E.g. in the static 4PN sector (i.e.  $G_N^5$ ) one meets

$$\begin{aligned}
 & \text{Diagram} = -i (8\pi G_N)^5 \left( \frac{(d-2)}{(d-1)} m_1 m_2 \right)^3 \\
 & \int_{k_{1,2,3,4}} \frac{N_{50}}{k_1^2 k_2^2 k_3^2 k_4^2 k_{12}^2 k_{34}^2 \hat{k}_{24}^2 p_{13}^2 \hat{p}_{14}^2} \\
 & \text{Diagram} = c_1 \text{Diagram} + c_2 \text{Diagram} + c_3 \text{Diagram} + c_4 \text{Diagram} + c_5 \text{Diagram}
 \end{aligned}$$

in terms of 4-loop self-energy diagrams in gauge theory

# Reduction in terms of master integrals

No new master integrals at 5PN, 4PN ones did it all

Foffa, Mastrolia, RS, Sturm '17



$$= \frac{e^{2\varepsilon\gamma_E}}{s^{2-2\varepsilon} (4\pi)^{4+2\varepsilon}} \left\{ \frac{1}{2\varepsilon^2} - \frac{1}{2\varepsilon} - 4 + \frac{\pi^2}{24} \right.$$

$$\left. - \varepsilon \left[ 9 - \pi^2 \left( \frac{13}{8} - \log 2 \right) - \frac{77}{6} \zeta_3 \right] + \mathcal{O}(\varepsilon^2) \right\}$$

Numerical result obtained via Summertime by Lee & Mingulov  
 analytic result via PSLQ algorithm, fitting transcendentals to numerical result

Confirmed up to  $\mathcal{O}(\varepsilon^0)$  by Damour, Jaradowski '18

# Summary: 2 body dynamics expansions (spin-less)

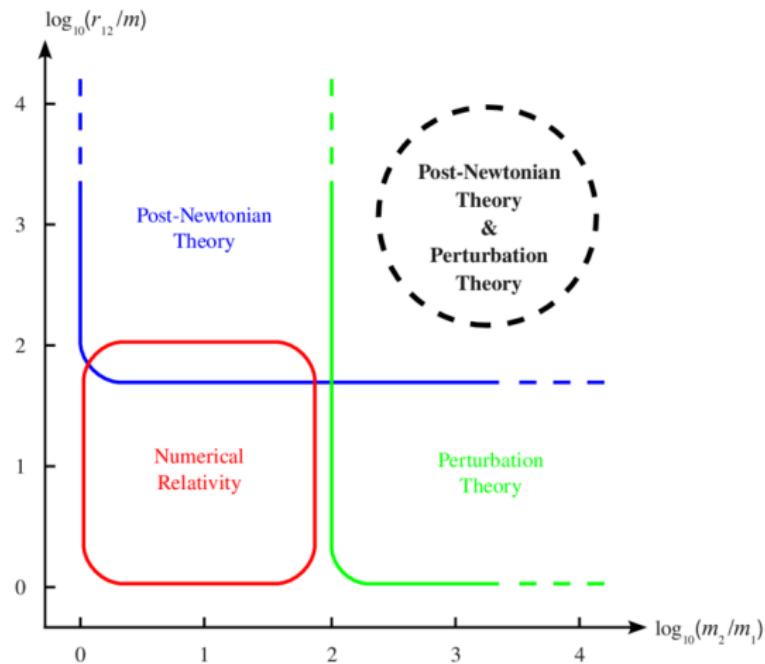
Post-Minkowskian expansion parameter is  $G_N M / r$ , vs PN expansion

$$\mathcal{L} = -Mc^2 + \frac{\mu v^2}{2} + \frac{GM\mu}{r} + \frac{1}{c^2} [\dots] + \frac{1}{c^4} [\dots]$$

Terms known so far

	N	1PN	2PN	3PN	4PN	5PN	...
0PM	1	$v^2$	$v^4$	$v^6$	$v^8$	$v^{10}$	$v^{12}$
1PM		$1/r$	$v^2/r$	$v^4/r$	$v^6/r$	$v^8/r$	$v^{10}/r$
2PM			$1/r^2$	$v^2/r^2$	$v^4/r^2$	$v^6/r^2$	$v^8/r^2$
3PM				$1/r^3$	$v^2/r^3$	$v^4/r^3$	$v^6/r^3$
4PM					$1/r^4$	$v^2/r^4$	$v^4/r^4$
5PM						$1/r^5$	$v^2/r^5$
...						$1/r^6(!)$	...

# Different approximation methods

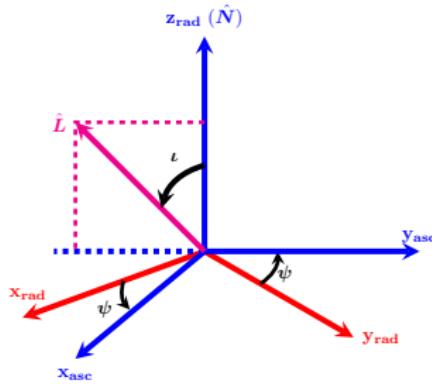
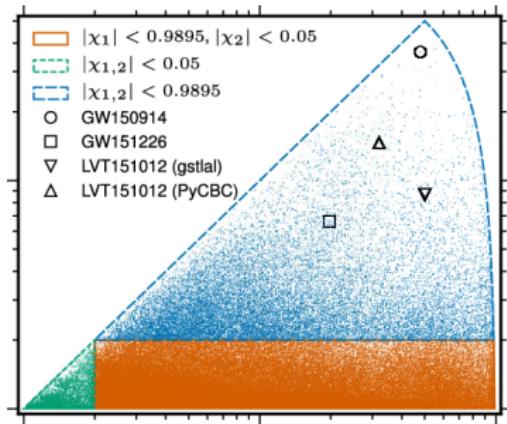


Blanchet et al.  
arXiv:1007.2614

Bini, Damour, Geralico in PRL ('19)+ completed 4PM dynamics from various input  
 Numerical relativity solution are expensive for large separation (large orbital scale) and  
 large mass ratios (long dynamical evolution time)

# Template bank and extrinsic parameters

Matched filter:  $MF(h) \equiv 2 \int_0^\infty \frac{(\tilde{h}_d(f)\tilde{h}_d^*(f) + \tilde{h}_d(f)^*\tilde{h}_d(f))}{S_n(f)} fd \log f$ ,  $SNR(h) = \sqrt{MF(h, h)}$



LIGO/Virgo, PRX (2016) 4, 041015,  
arXiv:1606.04856

J. Mendonça, RS, PRD (2023),  
arXiv:2302.03676

Parameters:  $\underbrace{m_1, m_2, \vec{\chi}_1, \vec{\chi}_2}_{intrinsic}, \underbrace{t, D_L, \overbrace{\theta, \alpha}^{sky loc.}, \overbrace{\psi, \iota, \phi}^{Euler angles}}_{extrinsic}$

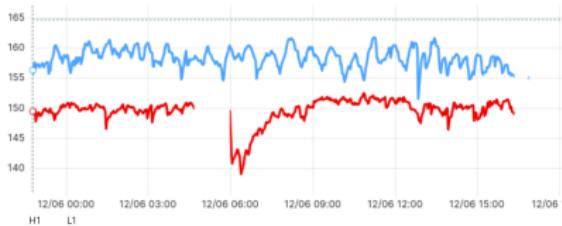
# Distance reach

Gravitational Wave Detector Network

Operational Snapshot as of Dec. 7, 2023 02:09:27 UTC

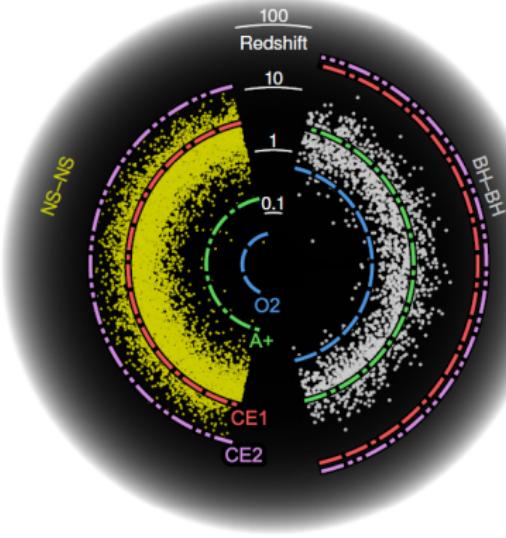
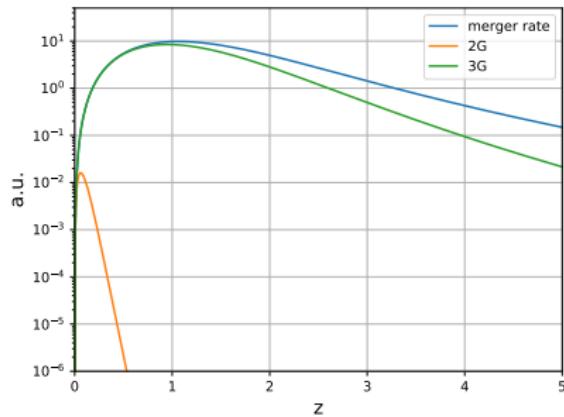
Detector	Status	Duration [hh]
GEO600	Unlocked	01:28
LIGO Hanford	Observing	03:04
LIGO Livingston	Observing	02:50
Virgo	Down	>99:00
KAGRA	No data	

GstLAL Inspiral Detector Range History (Mpc)



<https://online.igwn.org/>

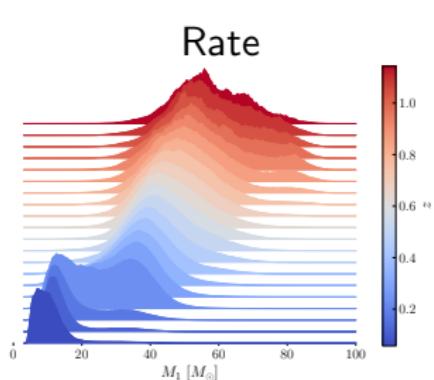
# How many more?



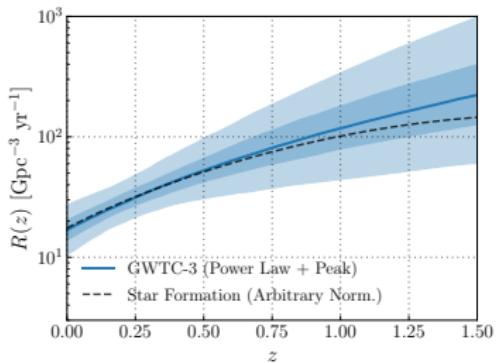
Leandro, Marra, RS PRD '21

arXiv:1903.04615

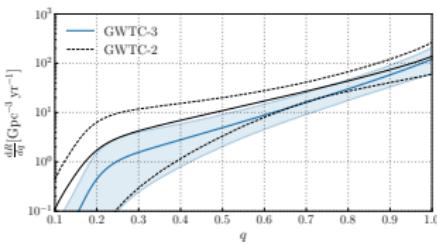
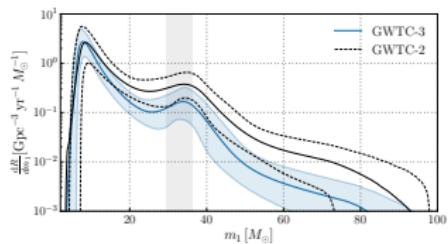
# What have we learnt?



Rinaldi+ 2310.03074



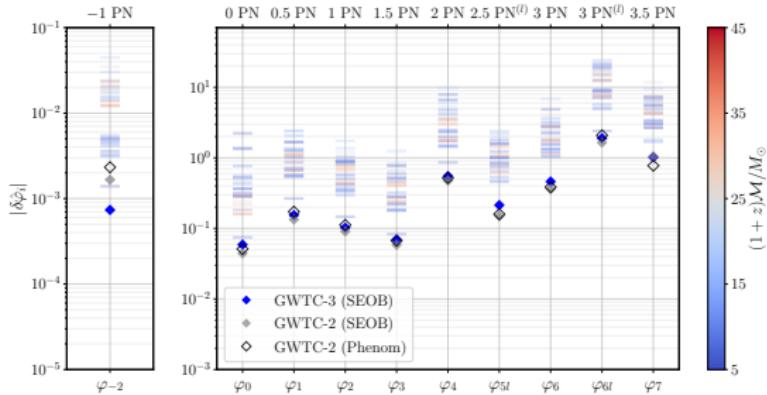
LIGO/Virgo/KAGRA arxiv:2111.03634



# Testing GR

Within the PN parametrization of the GWform phase ( $v^3 \equiv \pi GMf$ ):

$$\tilde{\psi}(f) = \frac{3}{128\eta v^5} \left[ \frac{\delta\phi_{-2}}{v^2} + (1 + \delta\phi_0) + \delta\phi_1 v + (1 + \delta\phi_2) v^2 + \dots \right]$$



LIGO/Virgo/KAGRA arXiv:2112.06861

Better constraints than binary pulsars (apart from  $\delta\phi_0 \lesssim 10^{-5}$ , and  $\delta\phi_{-2}$ )

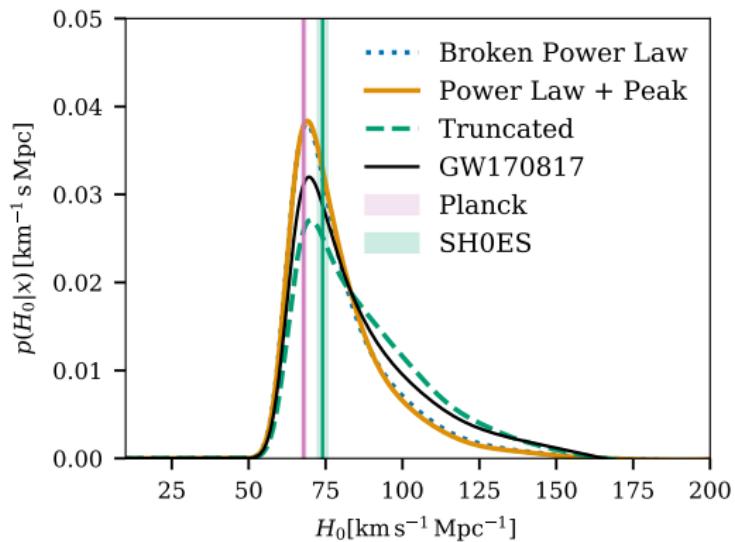
Nair, Yunes PRD (2020) arXiv:2002.02030

# Outline

- 1 Experiment
- 2 Observations (astro perspective)
- 3 What we actually get from data
- 4 Field theory methods for modeling binary systems
- 5 Cosmology
- 6 Summary

# The importance to know distance and redshift

Luminosity distance vs. redshift:  $D_L H_0 = z + O(z^2)$

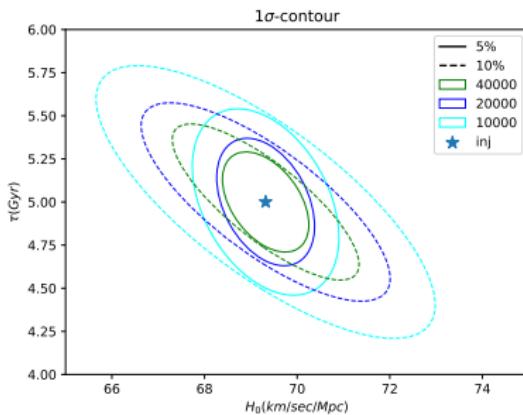
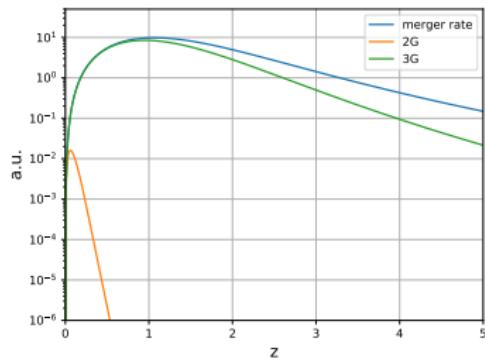


$H_0$  determination from EM bright 1 standard candle and 46 dark ones, short-circuiting with galaxy survey catalog GLADE+ Dálya et al. arXiv:2110.06184

LIGO/Virgo/KAGRA arXiv:2111.03604

# Black sirens

Information also stored in black sirens if *statistical distribution* of merger known (with hyper-parameter  $\tau$ )



Worst prior knowledge of the redshift distribution (modeling merger rate with more hyper-parameters) degrades predictive power of cosmo pars  
**Opportunity:** fit cosmology **and** population property

H. Leandro, V. Marra, RS PRD '21

# Outline

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# Summary

- Gravitational Wave Astronomy is a young and fast growing science, its impact will go beyond astronomy
- Field theory methods to solve the 2-body problem in GR are being used as efficient tools for computations from first-principle
- For future developments going to higher order will lead to new master integrals, stumbling block for any perturbative method (PN, PM...)
- Accuracy improvement expected for cosmological parameter/population properties