Particle acceleration: theory

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High energy astrophysics in the multi-messenger era
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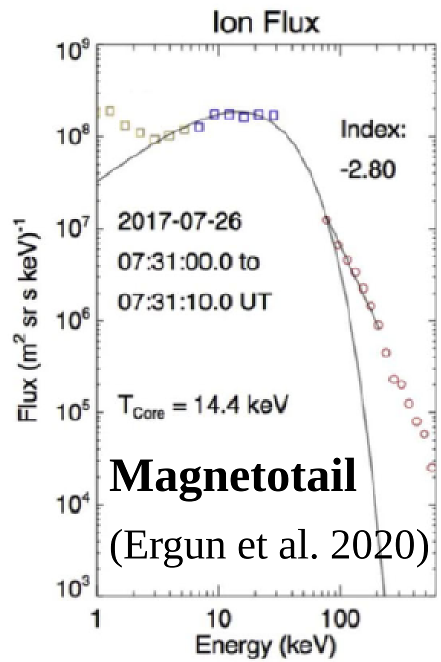
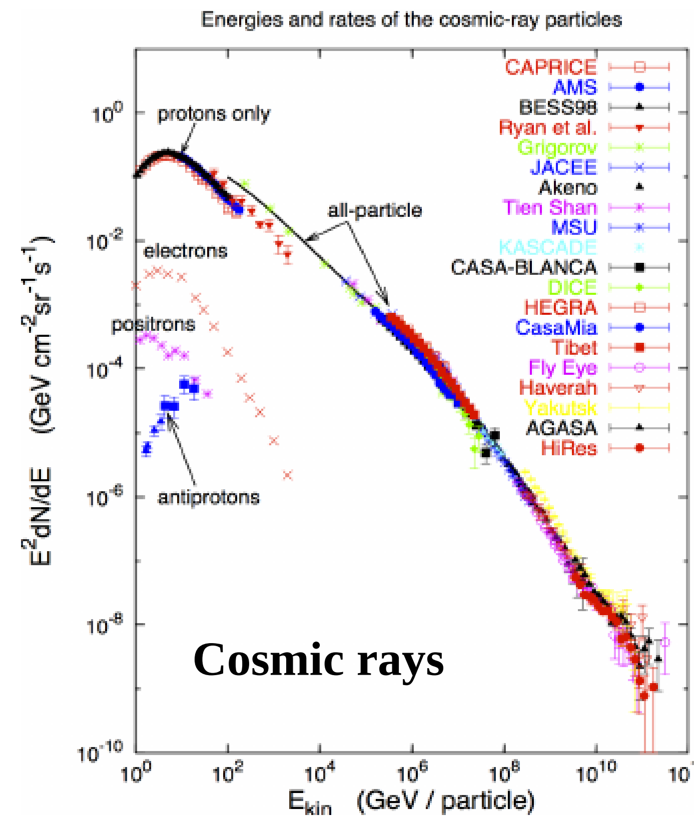
Program

- Overview of the basic and conceptual aspects of the following acceleration mechanisms:
  - Fermi acceleration in turbulence and non-relativistic shocks
  - Acceleration during magnetic reconnection

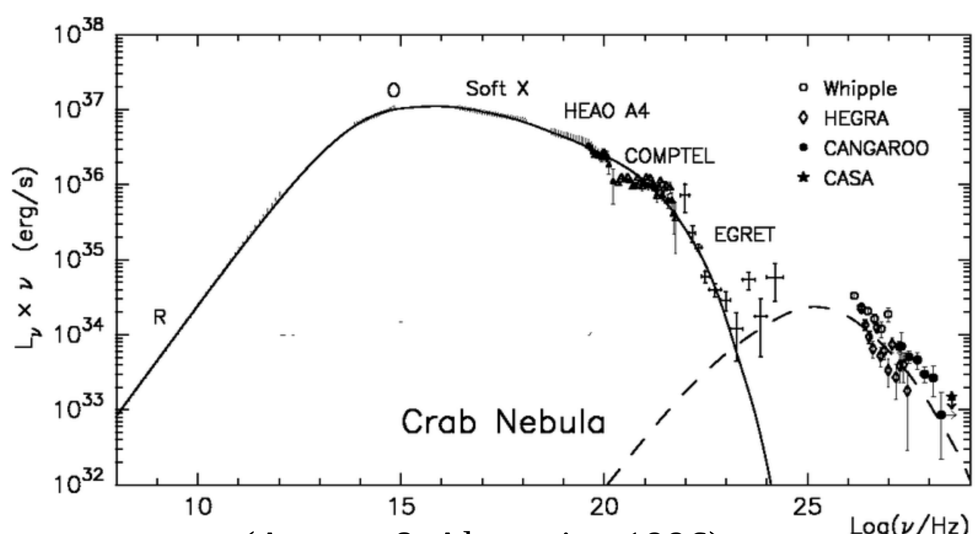
Evidences of non-thermal particles in the Universe

- Direct detection of non-thermal particles





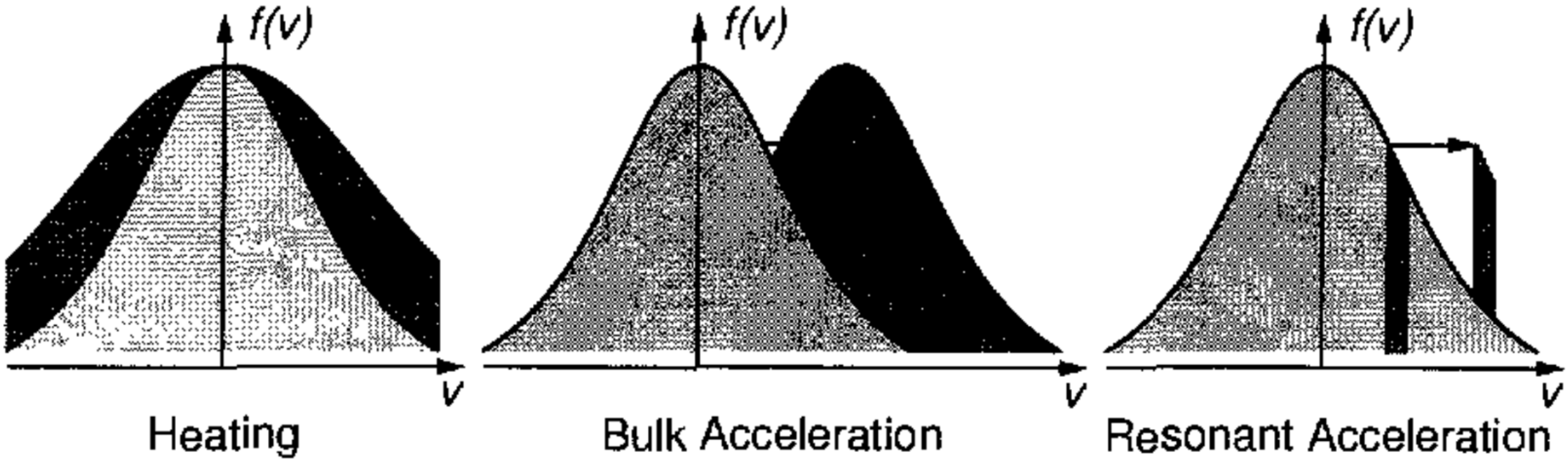
- Non-thermal radiation from astrophysical sources



(Atoyan & Aharonian 1996)

Particle energization: thermal vs. non-thermal

Velocity distribution function f(v):



(Treumann & Baumjohann 1997)

- We will focus on the generation of non-thermal distributions

Acceleration channels

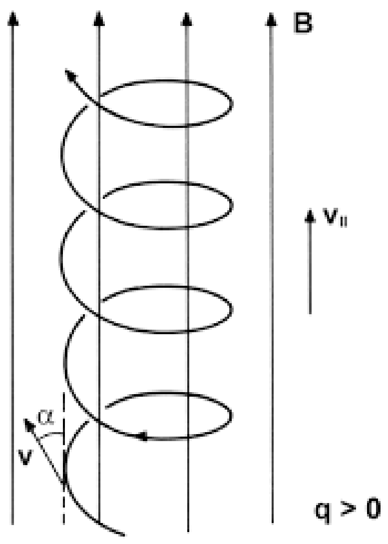
- Charged particles → electromagnetic mechanisms
- Explosive energy-release phenomena in space, solar, and astrophysical environments
- Shocks:
  - Earth's bow shock, interplanetary shocks triggered by CME, SNR, stellar winds, radio lobes of the AGN jets, intracluster medium of galaxies
- Magnetic reconnection:
  - Earth's magnetosphere, solar and stellar flares, magnetized AGN jets, accredtion disks, pulsar magnetosphere, wind, PNWe

Dynamics of charged particle in EM fields

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma m \mathbf{v}) = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right)$$

- In the absence of electric field: $$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma m \mathbf{v}) = q\frac{\mathbf{v}}{c} \times \mathbf{B}$$
  - Gyro-motion

$$\Omega = \frac{qB}{\gamma mc} \qquad R_L = \frac{v_\perp}{\Omega} = \frac{\gamma m v_\perp c}{qB}$$



- Magnetic fields do not perform work:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma m c^2) = q\mathbf{v} \cdot \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) = q\mathbf{v} \cdot \mathbf{E}$$

  - **Electric field required for acceleration**

Injection into the acceleration mechanism

The rate of particle energy gain must be faster than the rate of cooling:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \alpha E - \tau_{\mathrm{cool}}^{-1} E$$

Evolution of the distribution N(**r**, E, t):

$$\frac{\mathrm{d}N}{\mathrm{d}t} = D\nabla^2 N - \frac{\partial}{\partial E}\left\{(\alpha - \tau_{\mathrm{cool}}^{-1})EN\right\} - \frac{N}{\tau_{\mathrm{esc}}} + Q(E)$$

- τ_{cool} can define E_{MIN} and E_{MAX} of accelerated particles

Evolution of the distribution N(**r**, E, t):

$$\frac{\mathrm{d}N}{\mathrm{d}t} = D\nabla^2 N - \frac{\partial}{\partial E}\left\{(\alpha - \tau_{\mathrm{cool}}^{-1})EN\right\} - \frac{N}{\tau_{\mathrm{esc}}} + Q(E)$$

Stationary solution (no diffusion or injection), N(**r**, E, t) = N(E):

$$-\frac{\mathrm{d}}{\mathrm{d}E}\left\{(\alpha - \tau_{\mathrm{cool}}^{-1})EN\right\} - \frac{N}{\tau_{\mathrm{esc}}} = 0$$

Simple case when $\alpha\tau_{esc}$ and τ_{esc}/τ_{cool} have no dependence on E allows for a power-law solution:

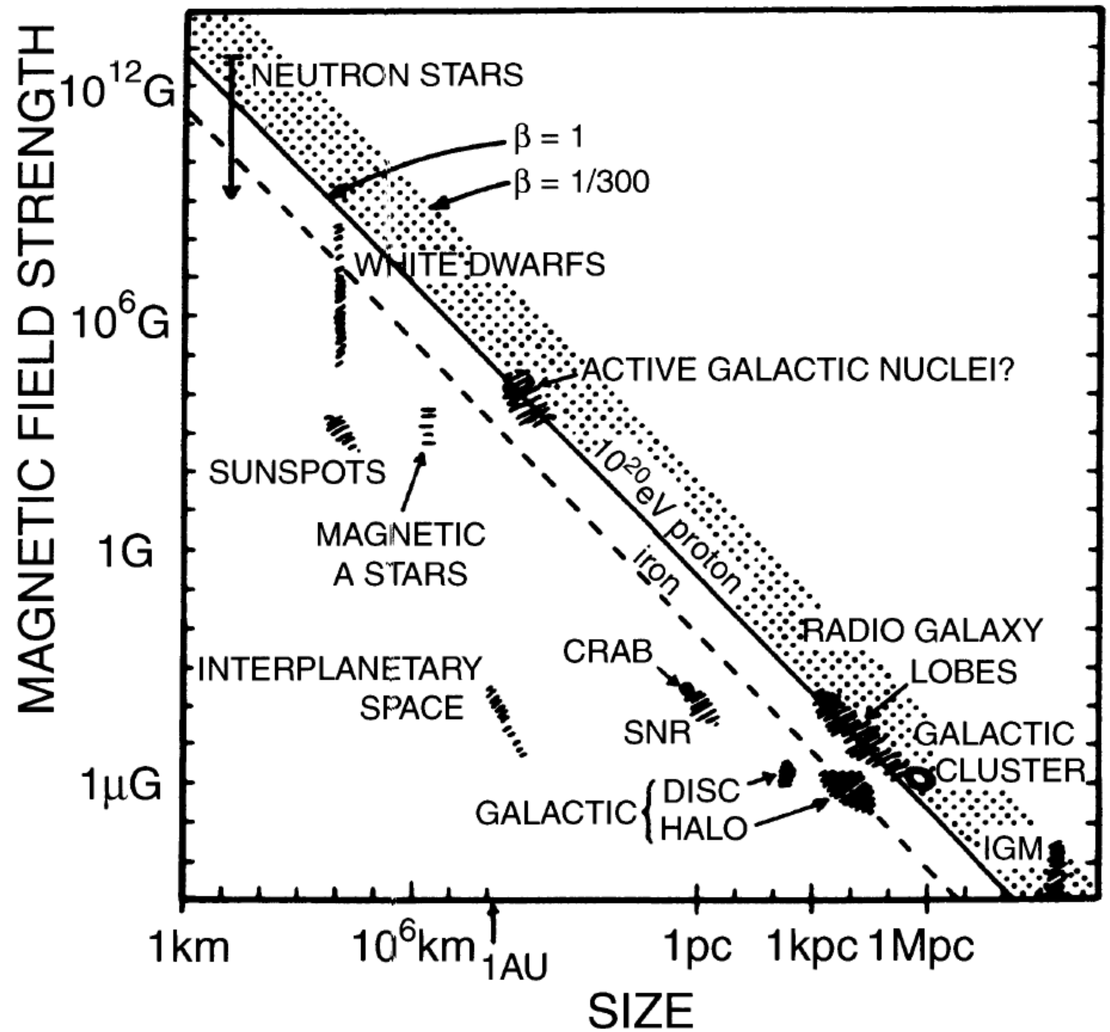
$$N(E) = KE^{-\Gamma}, \quad \Gamma = 1 + \frac{1}{(\alpha - \tau_{\mathrm{cool}}^{-1})\tau_{\mathrm{esc}}} + \frac{\partial \ln(\alpha - \tau_{\mathrm{cool}}^{-1})}{\partial \ln E}$$

Maximum energy: Hilla's diagram

$$R_L = \frac{\gamma m v_\perp c}{qB} \leq \; L$$

$$E_{\rm max} = \int zeE \, {\rm d}x = zeBUL$$

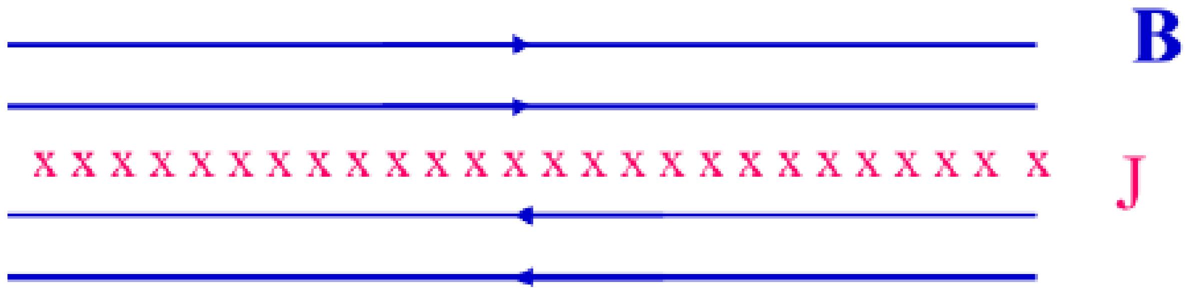
$$\frac{E_{\rm max}}{z\beta} = eBcL$$



Hillas 1984

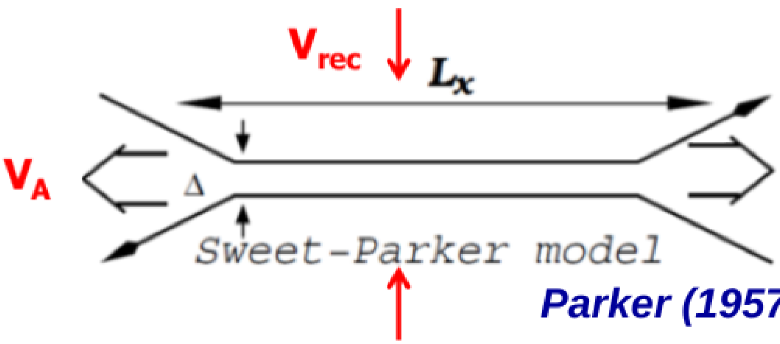
Acceleration in magnetic reconnection

- Electric field in the current sheet between two magnetized regions of opposite polarity
  - **Source of free energy**



Magnetic reconnection rate

Laminar model



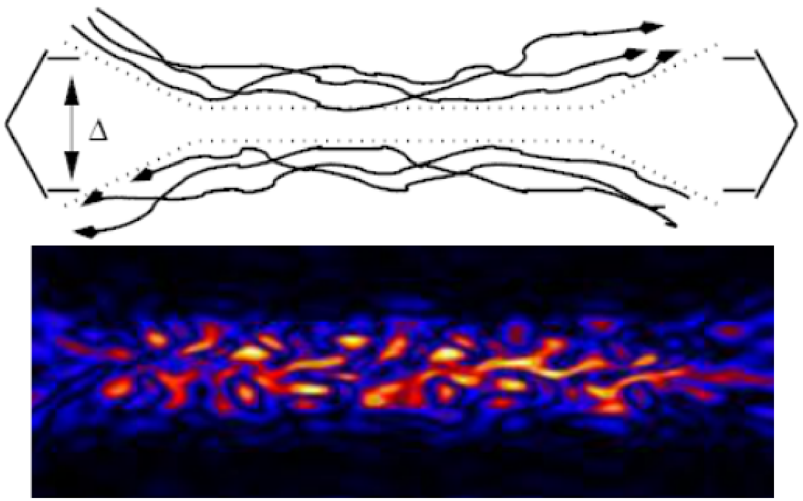
Parker (1957)

$$v_A = \frac{B}{\sqrt{4\pi\rho}}$$

$$v_{\rm rec} = v_A \left(\frac{L v_A}{\eta_{\rm Ohm}} \right)^{-1/2}$$

$v_{\rm rec} \ll v_A$ ⇒ "Slow"

Turbulent model

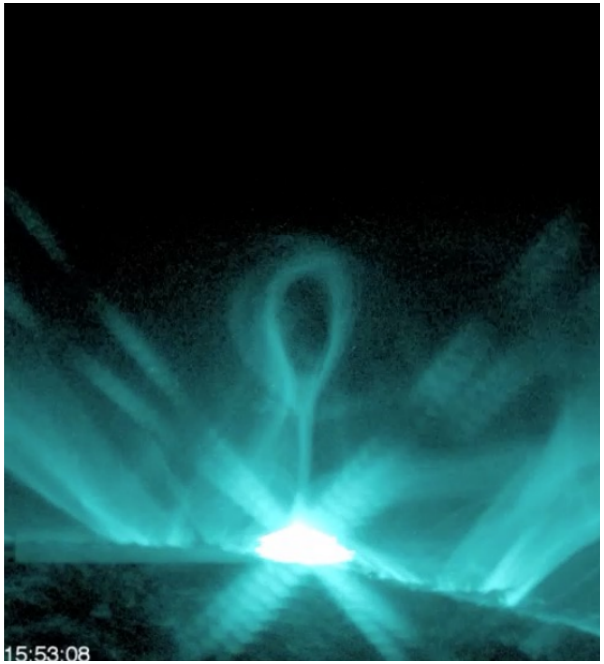


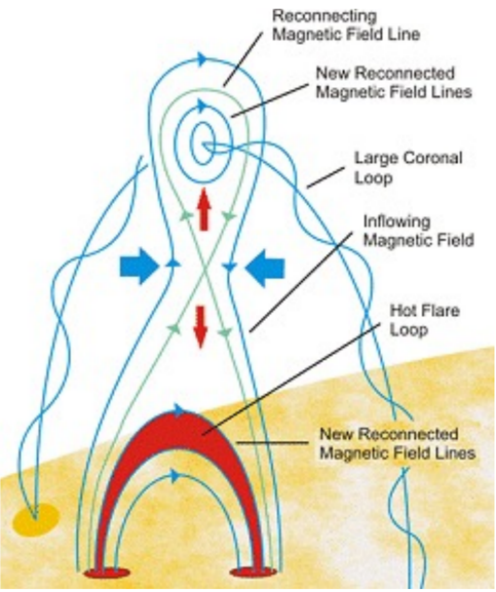
Lazarian & Vishniac (1999)
Kowal et al. (2009)

$$v_{\rm rec} = v_A \left(\frac{L v_A}{D_{\mathbf{B}}} \right)^{-1/2}$$

$v_{\rm rec} \sim v_A$ ⇒ "Fast"

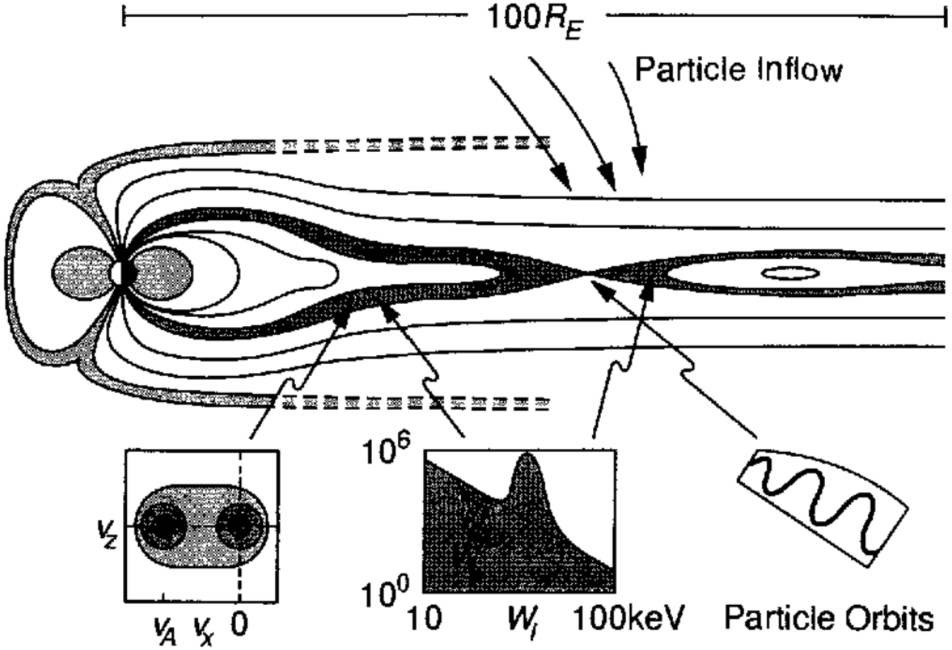
Solar flares





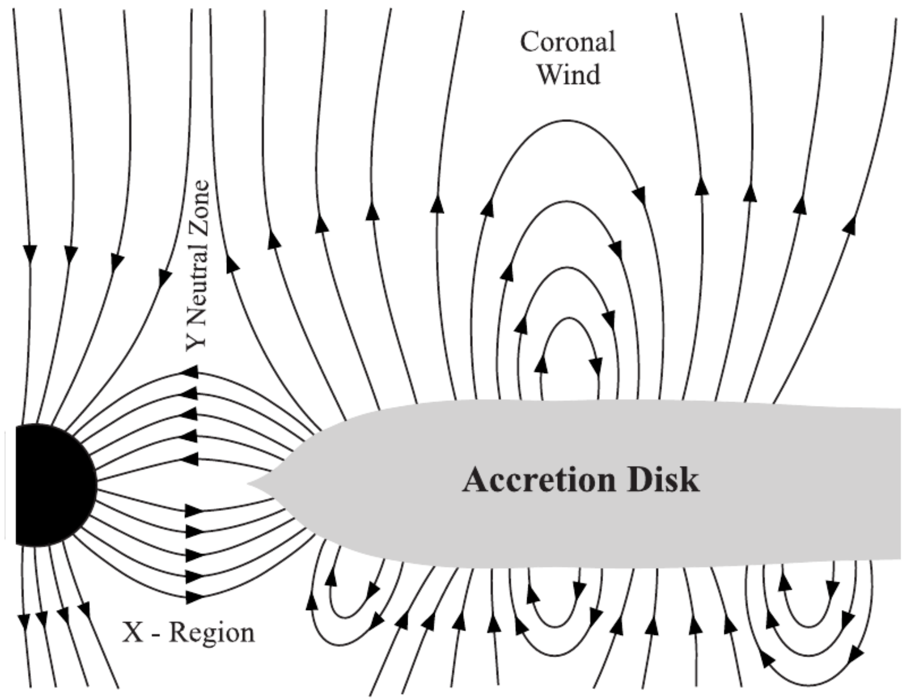
(Gordon Holman and NASA)

Magnetotail



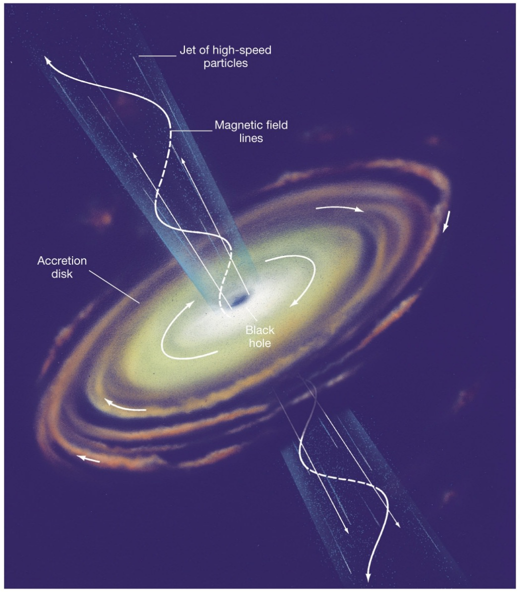
(Treumann & Baumjohann 1997)

Mini-quasar



(de Gouveia Dal Pino & Lazarian 2005)

AGN Jets



(Pearson Education)

Pulsar winds

(NASA)

Adiabatic energization

- Reversible, conservation of adiabatic invariant
- First adiabatic invariant (magnetic moment):

$$\mu = \frac{p_\perp}{2B} = \frac{\gamma m v_\perp}{2B}$$

- Energy evolution in the description of the particle's guiding center:

$$\frac{d\varepsilon}{dt} = q\mathbf{E}_\parallel \cdot \mathbf{v}_\parallel + \frac{\mu}{\gamma}\left(\frac{\partial B}{\partial t} + \mathbf{u}_\mathbf{E} \cdot \nabla B\right) + \gamma m_e v_\parallel^2 (\mathbf{u}_\mathbf{E} \cdot \boldsymbol{\kappa})$$

Electric drift vel: $\mathbf{u}_\mathbf{E} \doteq \mathbf{E} \times \mathbf{B}/B^2$ Field curvature: $\boldsymbol{\kappa} = \mathbf{b} \cdot \boldsymbol{\nabla}\, \mathbf{b}$

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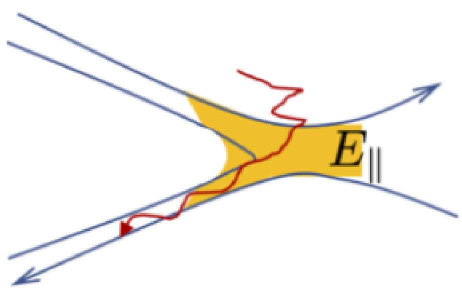
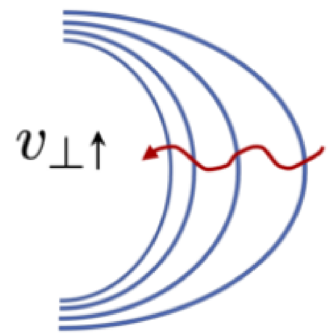
Direct acceleration



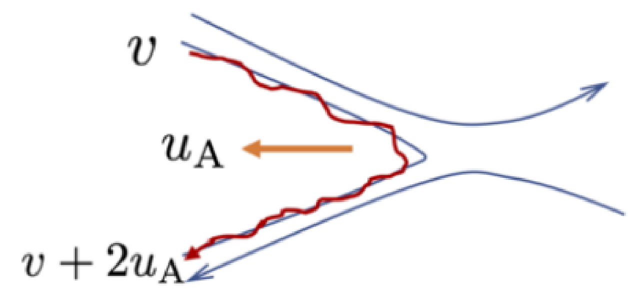
usually
less important

Betatron acceleration



increases
perpendicular energy

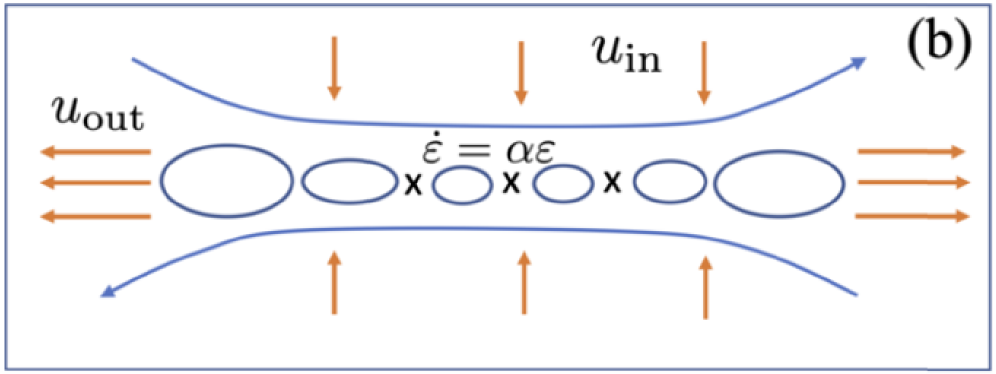
Fermi acceleration



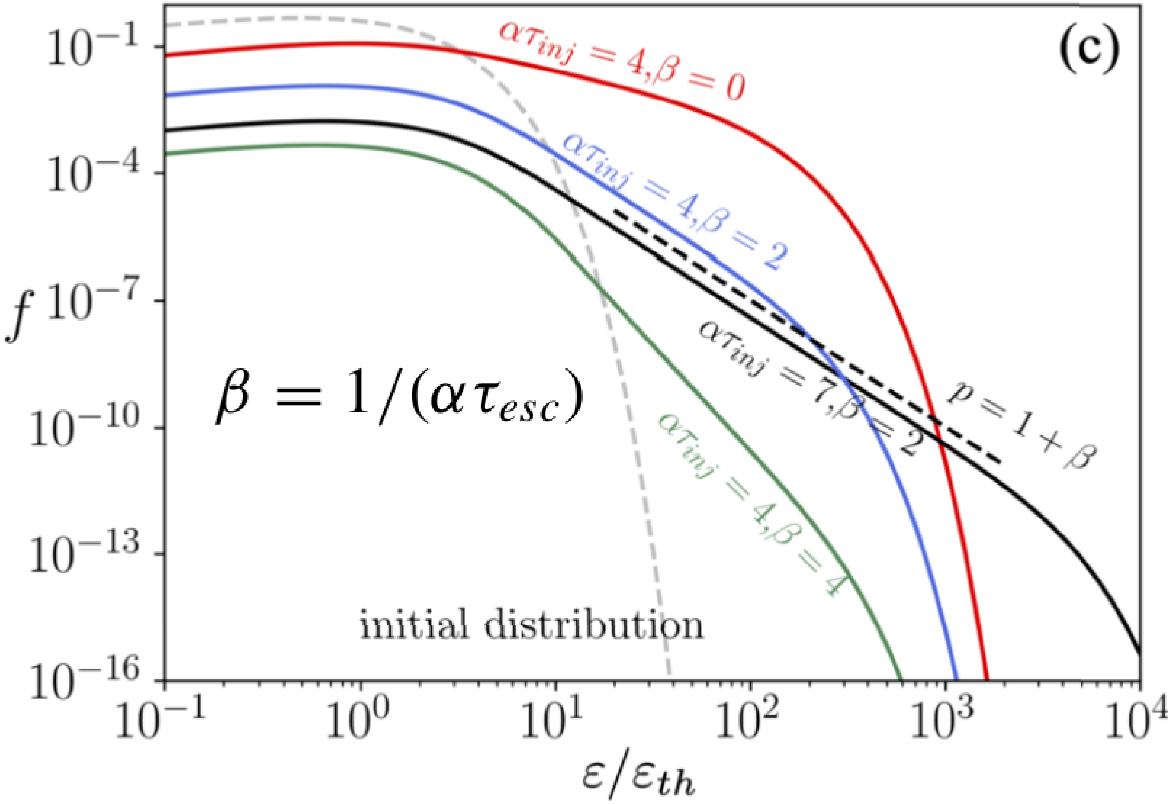
increases
parallel energy

(Figures: Oka et al. 2023)

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \varepsilon}(\dot{\varepsilon} f) = \frac{f_{inj}}{\tau_{inj}} - \frac{f}{\tau_{esc}}$$

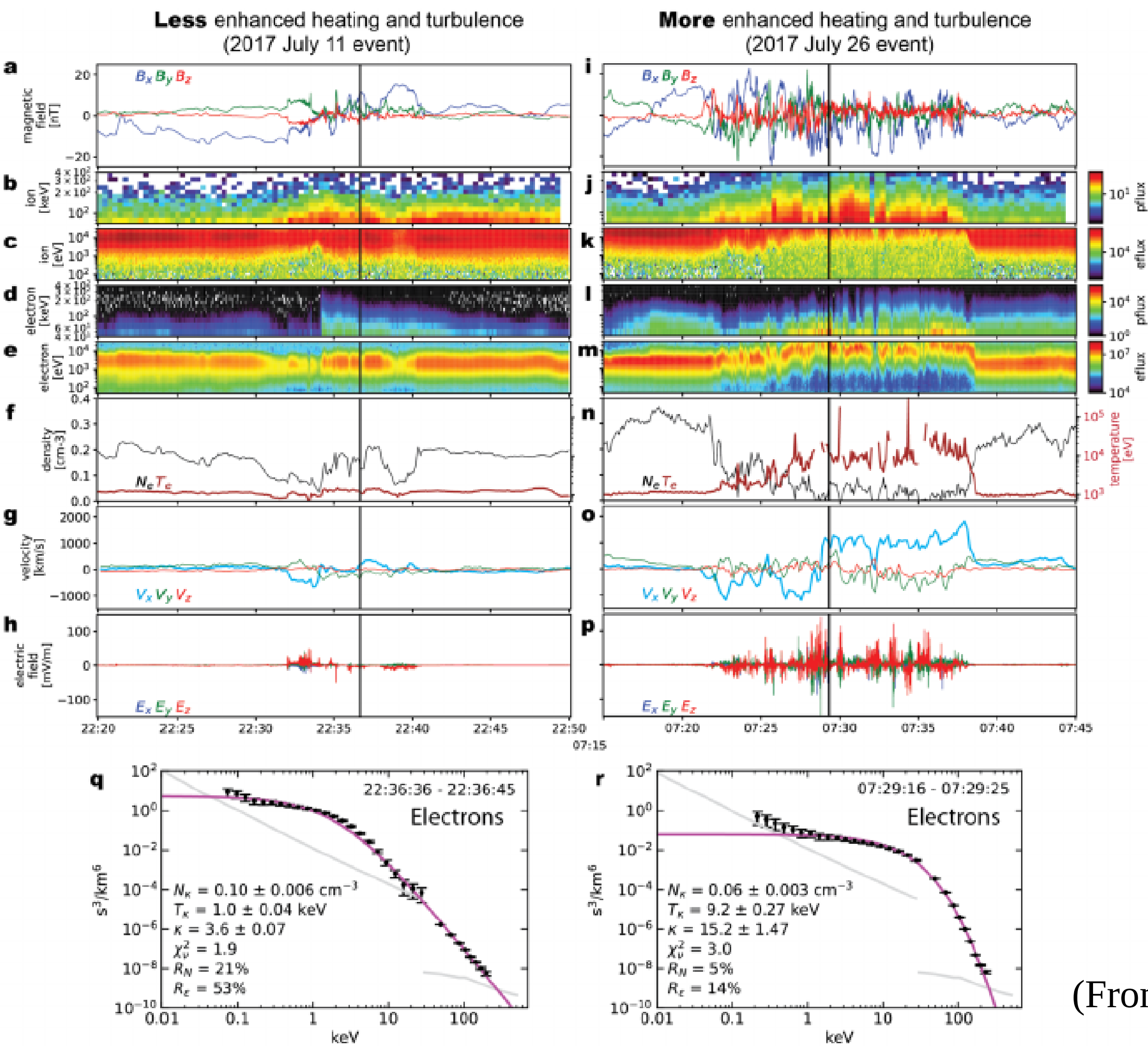


(Oka et al. 2023)



Particle acceleration during reconnection events in the magnetotail

Data: Magnetospheric Multi-Scale (MMS)



(From: Oka et al. 2023)

RMHD Simulations of Reconnection driven by Kink turbulence in Magnetically Dominated Relativistic Jets & Reconnection Particle Acceleration

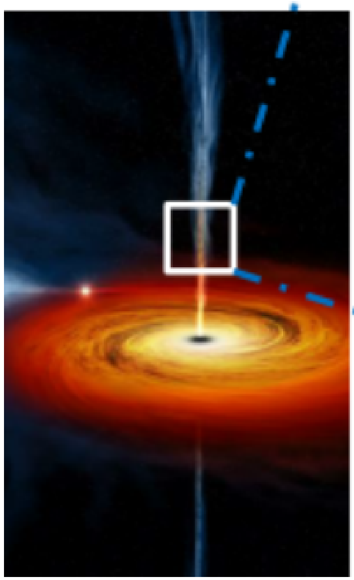
Precession perturbation causes lateral kink that distorts the column:

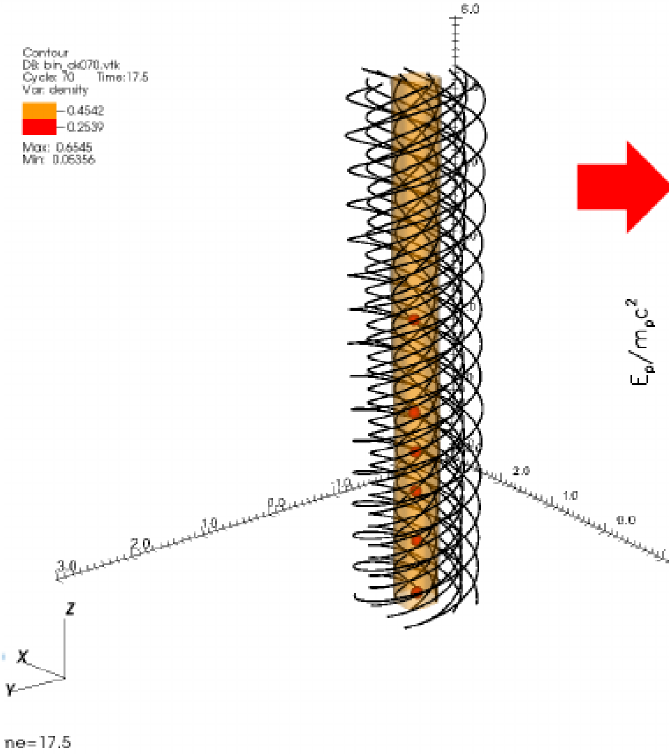
-> turbulence

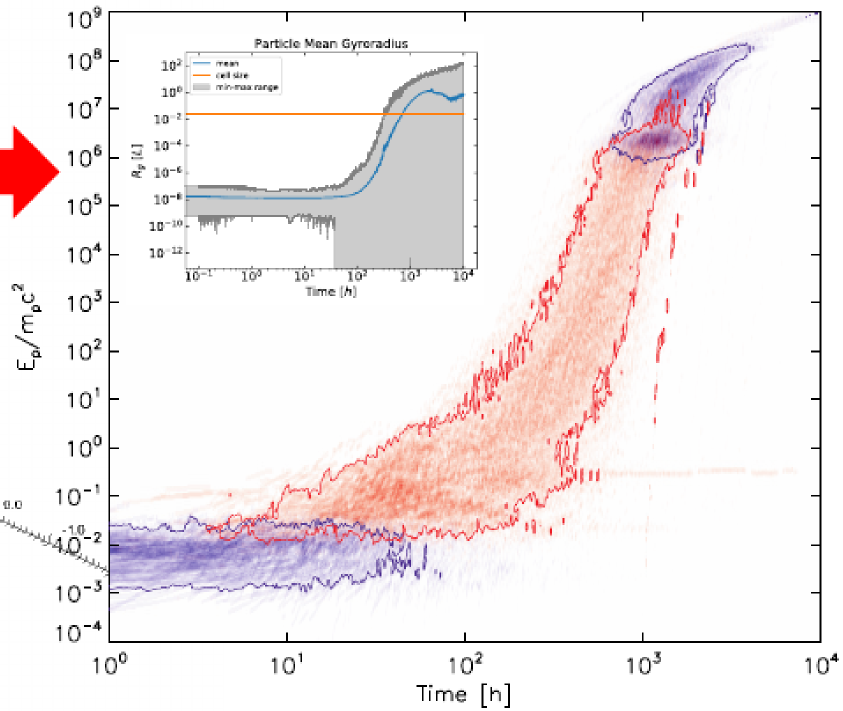
-> **Reconnection!**

-> **Acceleration!**

$$\sigma = B^2/\gamma^2 \rho h \ \sim 1$$







(de Gouveia Dal Pino et al. 2019)

(Kadowaki et al. 2021)

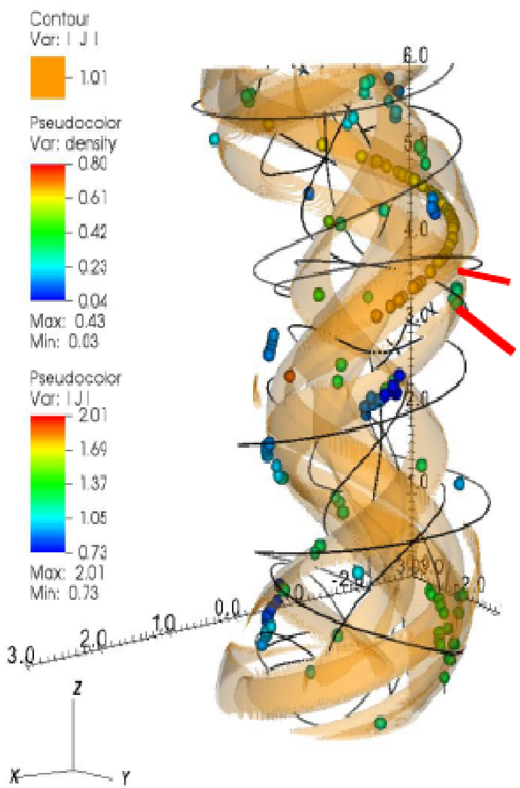
(Medina-Torrejón et al. 2021)

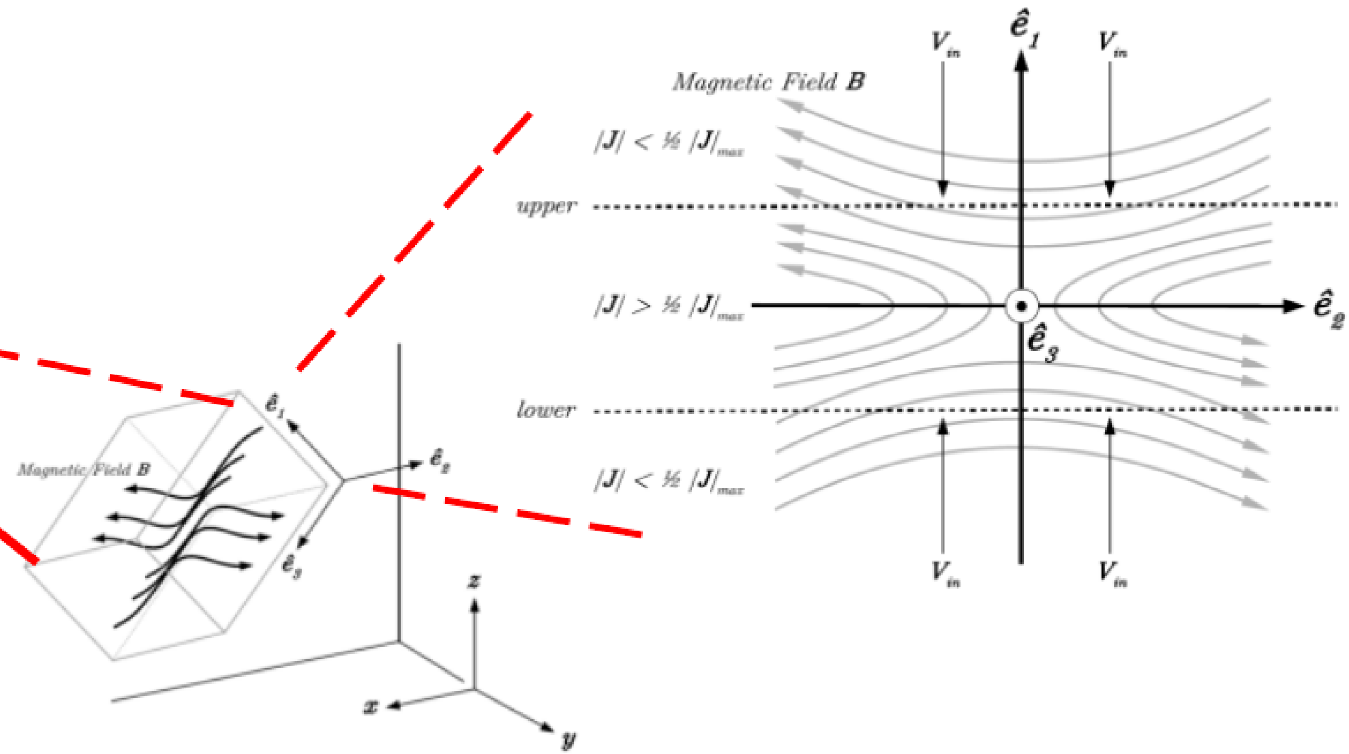
(Medina-Torrejón et al. 2023)

(credit: E. M. de Gouveia Dal Pino, IAG-USP)

Identification of Fast Reconnection driven by Kink in Relativistic Jets

- Search algorithm (Kadowaki et al. 2018) extended to 3D analysis





$$\left\langle \frac{V_{in}}{V_A} \right\rangle = \frac{1}{2} \left(\frac{V_{e_1}}{V_A}\bigg|_{lower} - \frac{V_{e_1}}{V_A}\bigg|_{upper} \right)$$

$$V_A = \sqrt{\frac{\left(B_{e_1}{}^2 + B_{e_2}{}^2 + B_{e_3}{}^2\right)}{\rho}}$$

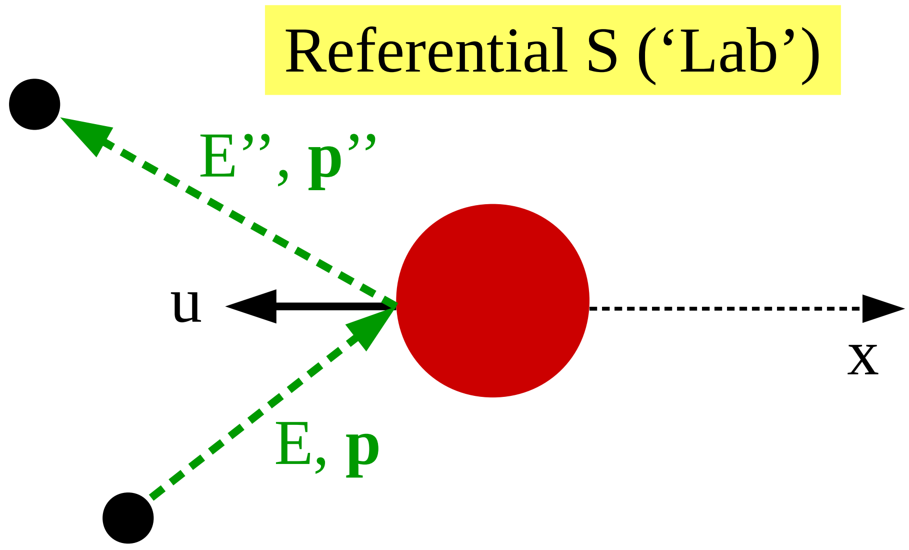
(Kadowaki et al. 2021)
(Medina-Torrejón et al. 2021)
(Medina-Torrejón et al. 2023)

(credit: E. M. de Gouveia Dal Pino, IAG-USP)

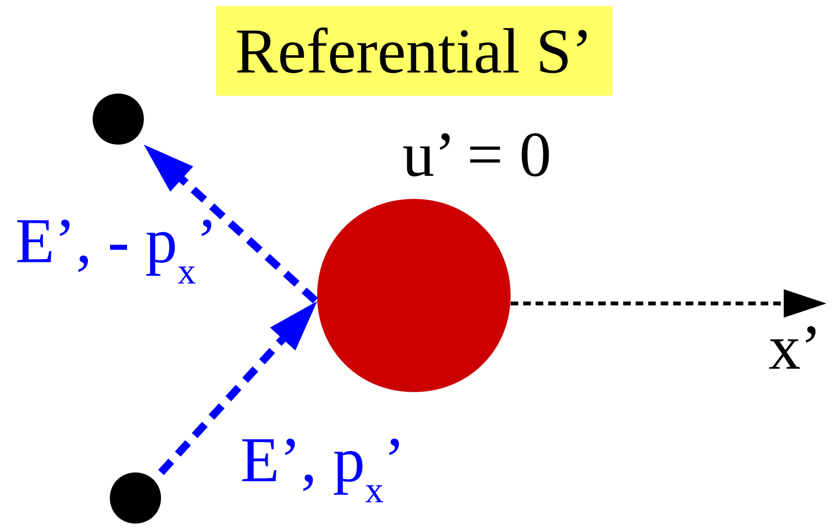
2^{nd} order diffusive Fermi acceleration

Particle initially with energy E and momentum **p** in the system S; cloud moving with velocity $\mathbf{u} = -u\,\hat{\mathbf{e}}_x$

$$p_x = p\mu \quad -1 < \mu < +1$$

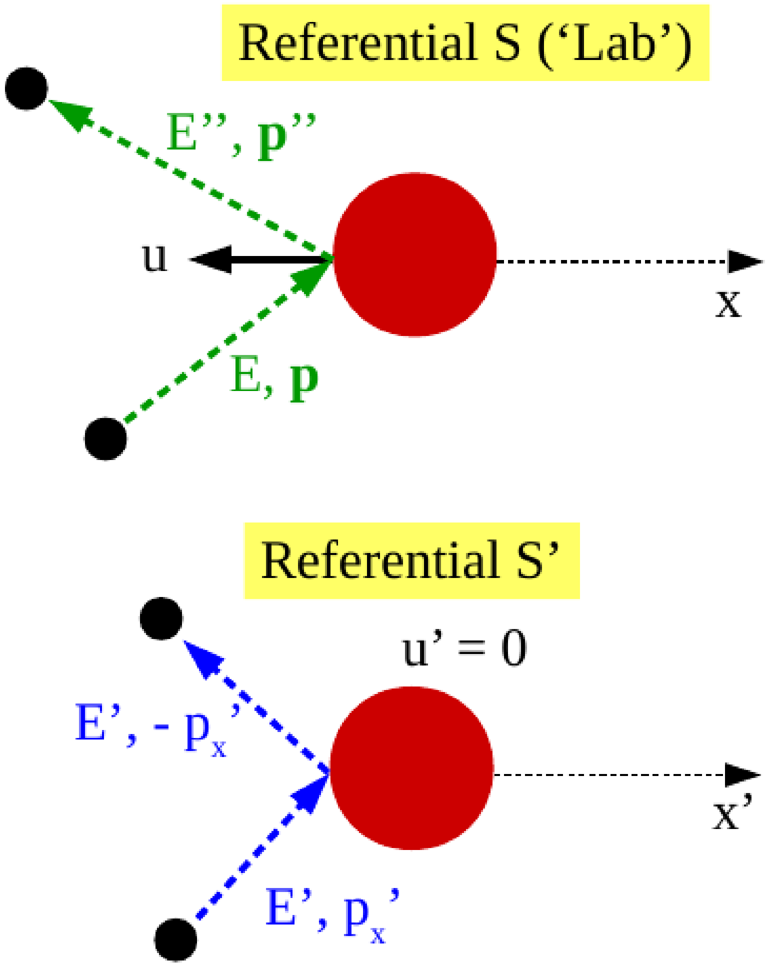


Lorentz transformation to frame S' moving with velocity u where cloud is at rest:



2^{nd} order diffusive Fermi acceleration

Lorentz transformation to frame S' moving with velocity $\mathbf{u} = - u\, \hat{\mathbf{e}}_x$ where cloud is at rest:



$$E' = \gamma(E + \beta c p_x) = \gamma(E + \beta c p \mu)$$

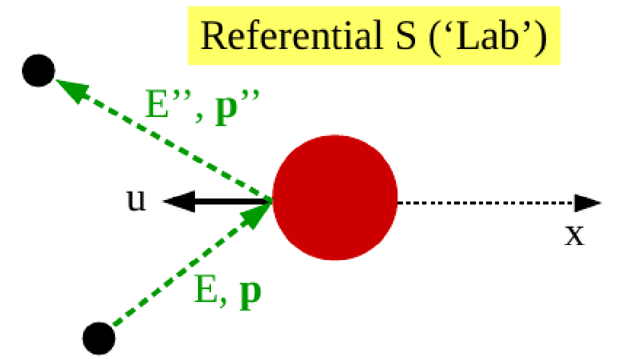
$$p'_x = \gamma(p_x + \beta E/c) = \gamma(p\mu + \beta E/c)$$

$$\gamma = \frac{1}{(1 - u^2/c^2)^{1/2}}$$

$$\beta = u/c$$

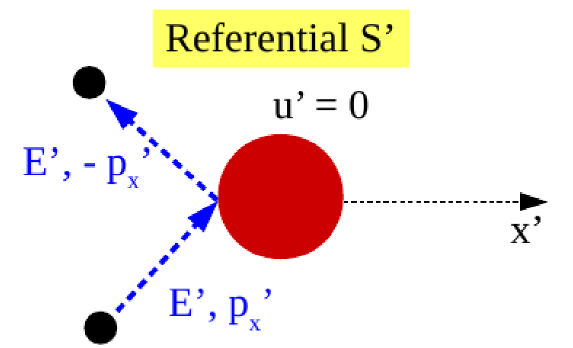
Upon elastic collision in S': $p'_x \rightarrow -p'_x$

New energy in S after scattering: $E'' = \gamma \left\{ E' - \beta c(-p'_x) \right\}$



$$E' = \gamma(E + \beta c p_x) = \gamma(E + \beta c p \mu)$$

$$p'_x = \gamma(p_x + \beta E/c) = \gamma(p\mu + \beta E/c)$$



$$E'' = \gamma\left\{E' - \beta c(-p'_x)\right\} = \gamma^2 E\left(1 + \beta^2 + \frac{2\beta c p \mu}{E}\right)$$

$$= \gamma^2 E\left(1 + \beta^2 + 2\beta v \mu / c\right)$$

$$\frac{p}{E} = \frac{v}{c^2}$$

$$\gamma^2 = \frac{1}{(1 - \beta^2)} = 1 + \beta^2 + O(\beta^4)$$

$$\frac{E'' - E}{E} = \frac{\Delta E}{E} = \gamma^2\left(1 + \beta^2 + 2\beta v\mu/c\right) - 1$$

$$= \left(1 + \beta^2 + O(\beta^4)\right)\left(1 + \beta^2 + 2\beta v\mu/c\right) - 1$$

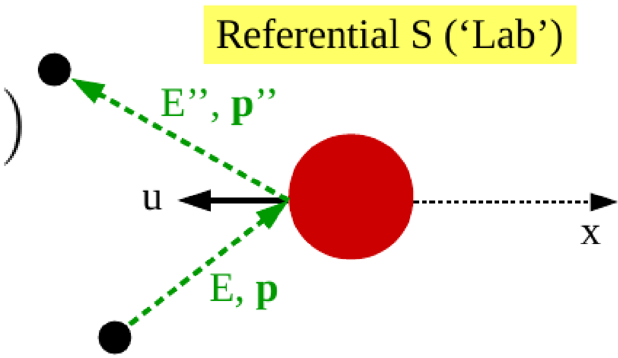
$$\frac{E'' - E}{E} = \frac{\Delta E}{E} = \gamma^2 \left(1 + \beta^2 + 2\beta v \mu / c\right) - 1$$
$$= \left(1 + \beta^2 + O(\beta^4)\right) \left(1 + \beta^2 + 2\beta v \mu / c\right) - 1$$

$$\frac{\Delta E}{E} = 2\beta^2 + 2\beta v \mu / c + O(\beta^3)$$

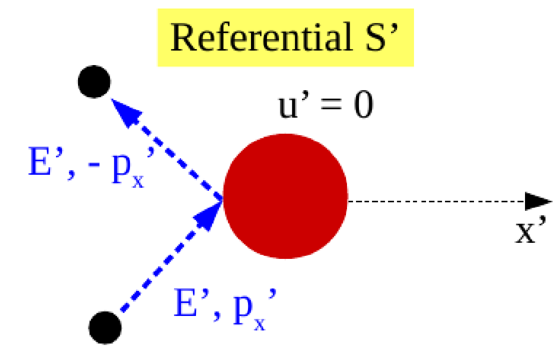
$$-1 < \mu < +1$$

Probability of scattering for different particle's directions:

$$P_{\mathrm{coll}}(\mu) \propto \tau_{\mathrm{coll}}^{-1}(\mu) P(\mu) \propto v_{\mathrm{approx}}(\mu) P(\mu)$$
$$v_{\mathrm{approx}}(\mu) = v + \beta c \mu \approx c(1 + \beta\mu)$$

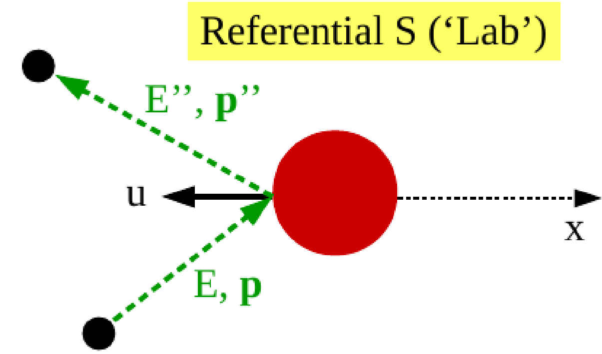


$$\Rightarrow \quad P_{\mathrm{coll}}(\mu) = \frac{1}{2}(1+\beta\mu), \quad \left(\int_{-1}^{+1} \mathrm{d}\mu P_{\mathrm{coll}}(\mu) = 1\right)$$

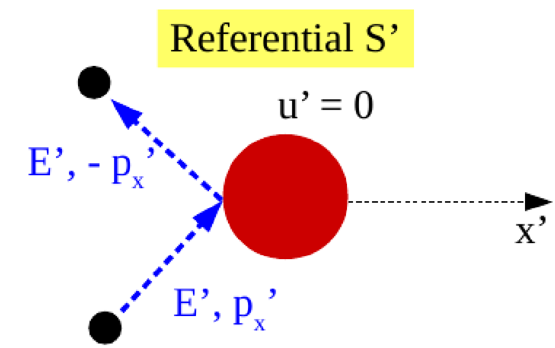


Observe that $P_{\mathrm{coll}}(\mu = +1) > P_{\mathrm{coll}}(\mu = -1)$

$$\frac{\Delta E}{E} = 2\beta^2 + 2\beta v\mu/c + O(\beta^3)$$



$$P_{\rm coll}(\mu) = \frac{1}{2}(1+\beta\mu), \quad \left(\int_{-1}^{+1} {\rm d}\mu P_{\rm coll}(\mu) = 1\right)$$



$$\left\langle \frac{\Delta E}{E} \right\rangle = \int_{-1}^{+1} {\rm d}\mu P_{\rm coll}(\mu) \left\{2\beta^2 + 2\beta v\mu/c\right\} = \frac{4}{3}\beta^2$$

Change in energy proportional to the second order in the Small parameter $\beta = u/c$

⇒ Fermi second order acceleration

$$\langle \Delta E \rangle = \frac{4}{3}\beta^2 E$$

$$\Rightarrow \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{4}{3}\beta^2 \tau_{\mathrm{scatt}}^{-1} E = \alpha E$$

$$(v/l) = \tau_{\mathrm{scatt}}^{-1}$$

l ~ distance between clouds

More generally: $$\frac{\mathrm{d}E}{\mathrm{d}t} = \alpha E - \tau_{\mathrm{cool}}^{-1} E$$

In the diffusion-loss equation for the distribution N(E):

$$\frac{\mathrm{d}N}{\mathrm{d}t} = D\nabla^2 N - \frac{\partial}{\partial E}\left\{(\alpha - \tau_{\mathrm{cool}}^{-1}) E N\right\} - \frac{N}{\tau_{\mathrm{esc}}} + Q(E)$$

In the diffusion-loss equation for the distribution N(E):

$$\frac{\mathrm{d}N}{\mathrm{d}t} = D\nabla^2 N - \frac{\partial}{\partial E}\left\{(\alpha - \tau_{\mathrm{cool}}^{-1})EN\right\} - \frac{N}{\tau_{\mathrm{esc}}} + Q(E)$$

Looking for the stationary solution (no diffusion or injection):

$$-\frac{\mathrm{d}}{\mathrm{d}E}\left\{(\alpha - \tau_{\mathrm{cool}}^{-1})EN\right\} - \frac{N}{\tau_{\mathrm{esc}}} = 0$$

Power-law solution:

$$N(E) = KE^{-\Gamma}, \quad \Gamma = 1 + \frac{1}{(\alpha - \tau_{\mathrm{cool}}^{-1})\tau_{\mathrm{esc}}} + \frac{\partial \ln(\alpha - \tau_{\mathrm{cool}}^{-1})}{\partial \ln E}$$

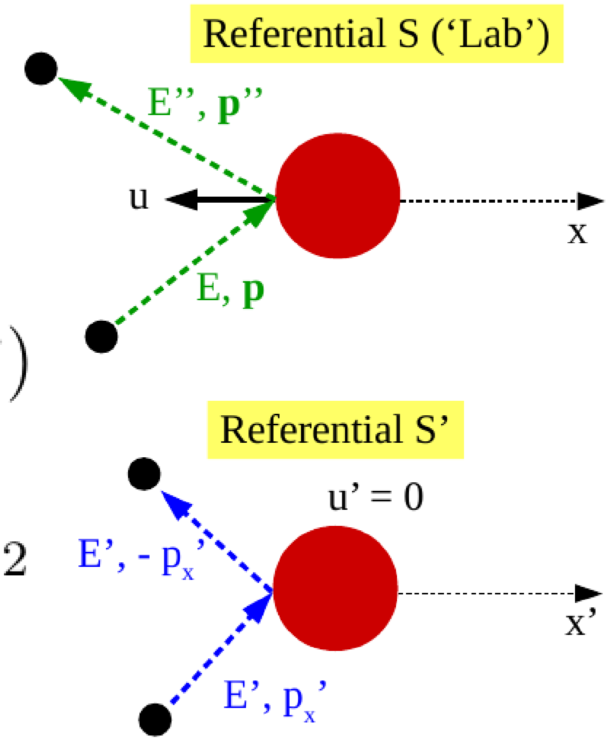
Adding the diffusion term in the energy space:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = D\nabla^2 N - \frac{\partial}{\partial E}\left\{(\alpha - \tau_{\rm cool}^{-1})EN\right\} - \frac{N}{\tau_{\rm esc}} + Q(E) + \boxed{\frac{1}{2}\frac{\partial^2}{\partial E^2}\left\{D_E N\right\}}$$

$$D_E = \frac{\mathrm{d}}{\mathrm{d}t}\left\langle (\Delta E)^2 \right\rangle$$

$$\frac{E'' - E}{E} = \Delta E = 2\beta^2 E + 2\beta v\mu/cE + O(\beta^3)$$

$$(\Delta E)^2 = 4\beta^2 v^2\mu^2/c^2 E^2 + O(\beta^3) \approx 4\beta^2\mu^2 E^2$$



$$(\Delta E)^2 = 4\beta^2 v^2 \mu^2 / c^2 E^2 + O(\beta^3) \approx 4\beta^2 \mu^2 E^2$$

Repeating the average over the probability for the collision direction,

$$P_{\rm coll}(\mu) \propto \tau_{\rm coll}^{-1}(\mu) P(\mu) \propto v_{\rm approx}(\mu) P(\mu) \quad \Rightarrow \quad P_{\rm coll}(\mu) = \frac{1}{2}(1+\beta\mu)$$

$$\langle (\Delta E)^2 \rangle = \int_{-1}^{+1} {\rm d}\mu P_{\rm coll}(\mu) \left\{ 4\beta^2 \mu^2 E^2 \right\} = \frac{4}{3}\beta^2 E^2$$

$$D_E = \frac{\rm d}{{\rm d}t} \langle (\Delta E)^2 \rangle = \frac{4}{3}\beta^2 \tau_{\rm scatt}^{-1} E^2$$

2^{nd} order Fermi acceleration: turbulence

Diffusion in the energy space can be related to the diffusion in the momentum space:

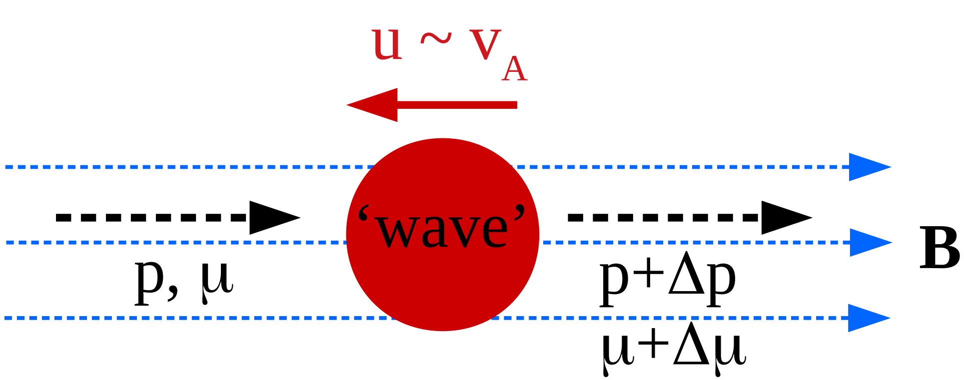
$$D_E \leftrightarrow D_{pp} \qquad D_{pp} = \frac{\mathrm{d}}{\mathrm{d}t}\left\langle (\Delta p)^2 \right\rangle \qquad D_{pp} \sim \beta^2 \tau_{\mathrm{scatt}}^{-1} p^2$$

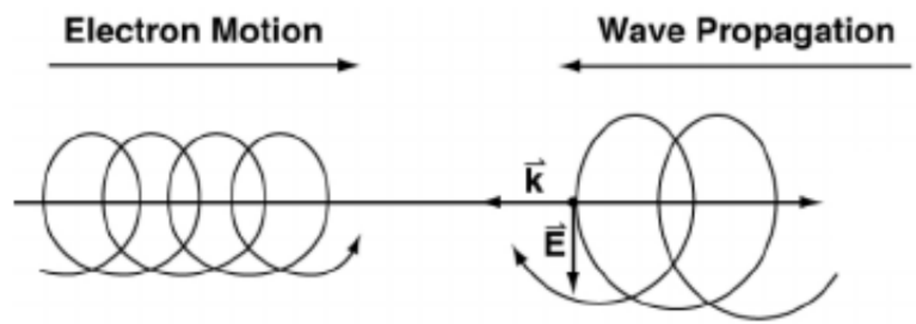
For turbulence plasma waves:

$$\tau^{-1}_{\mathrm{scatt,QLT}} \sim \left(\frac{\delta B_{\mathrm{res}}}{B_0}\right)^2 \Omega$$

Ressonance condition:

$$\omega - k_{\parallel} v_{\parallel} = n\Omega, \quad n = 0, 1, 2, 3, \ldots$$





Diffusion in the energy/momentum of the CR distribution function f(p,t) describes the 2nd order Fermi acceleration:

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial t}\left(p^2 D_{pp}\frac{\partial f}{\partial t}\right) - \frac{f}{\tau_{\rm esc}}$$

Allows a power-law for stationary solution:

$$D_{pp} \propto p^q \quad \Rightarrow \quad f(p) \propto p^{-\Gamma}$$

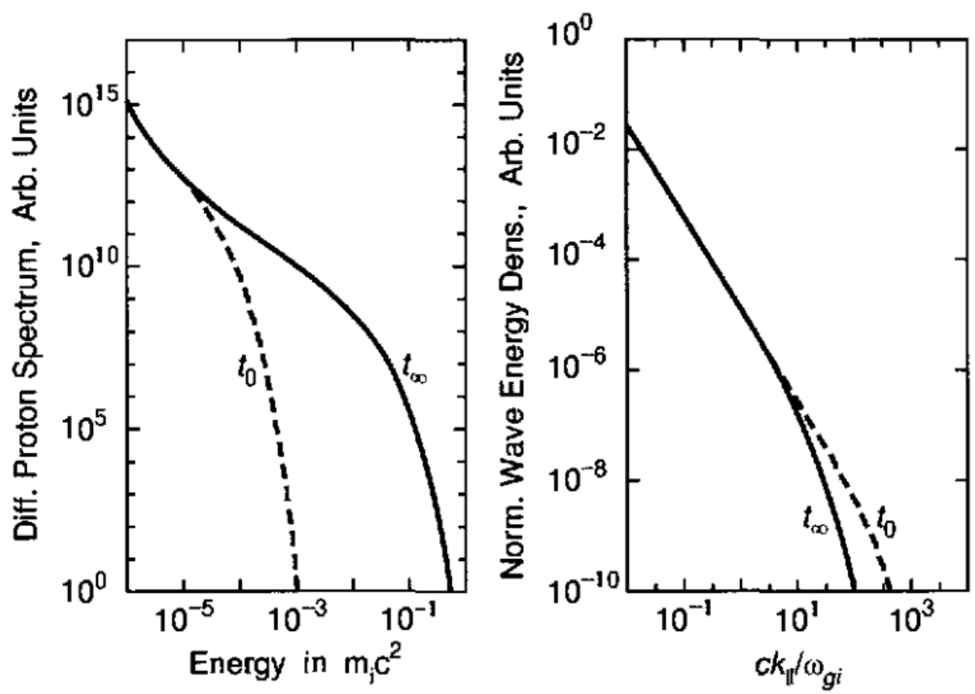
$$\Gamma = \frac{q+1}{2} + \left\{\left(\frac{q+1}{2}\right)^2 + \frac{\tau_{\rm acc}}{\tau_{\rm esc}}\right\}^{1/2} \qquad \tau_{\rm acc} = \frac{p^2}{D_{pp}}$$

assumption: $\dfrac{\tau_{\rm acc}(p)}{\tau_{\rm esc}(p)} = \mathrm{const}$

Diffusion in the energy/momentum of the CR distribution function f(p,t) describes the 2nd order Fermi acceleration:

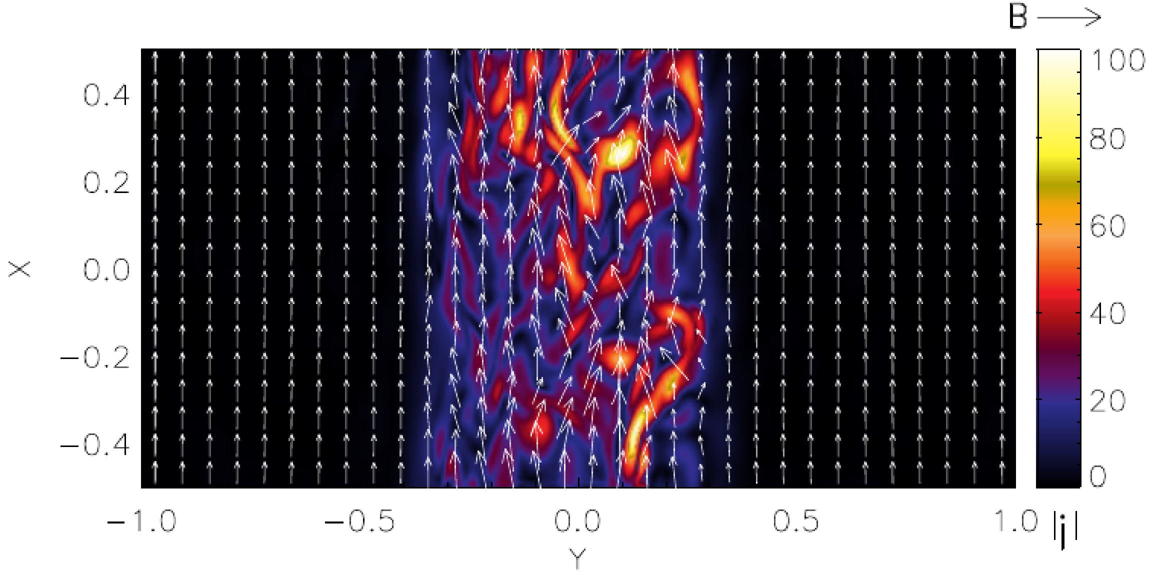
$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial t}\left(p^2 D_{pp}\frac{\partial f}{\partial t}\right) - \frac{f}{\tau_{\rm esc}}$$

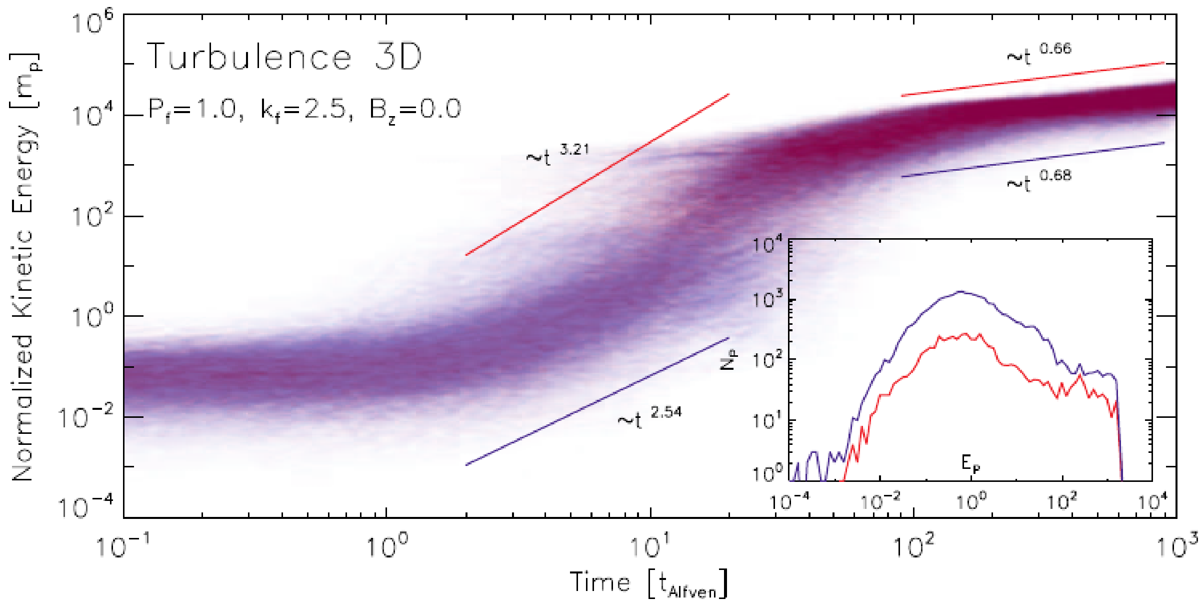
Protons accelerated in a spectrum o MHD waves:



(Treumann & Baumjohann 1997)

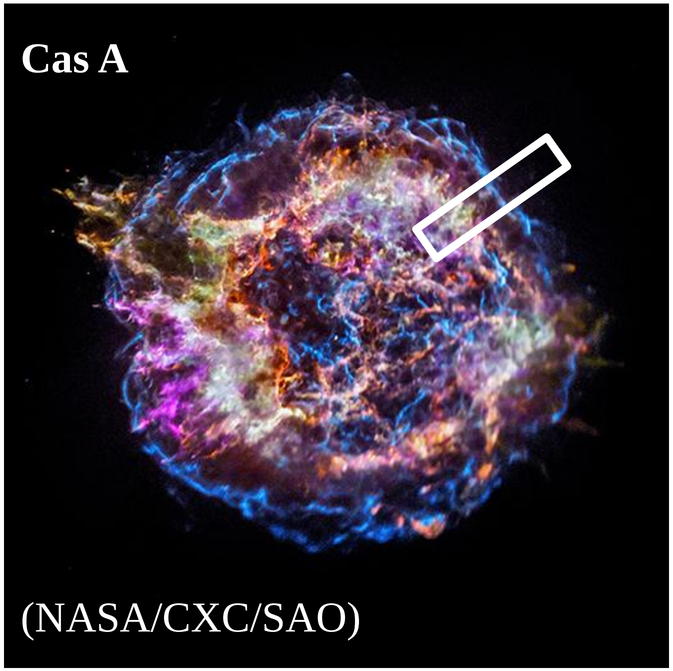
Test particles in MHD turbulence



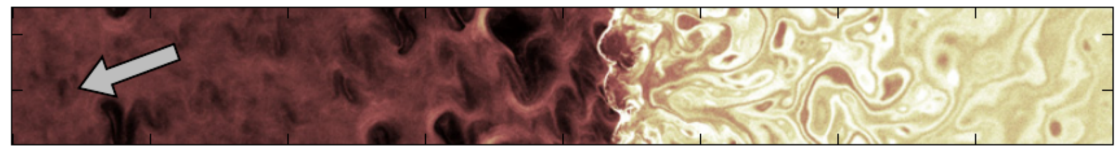


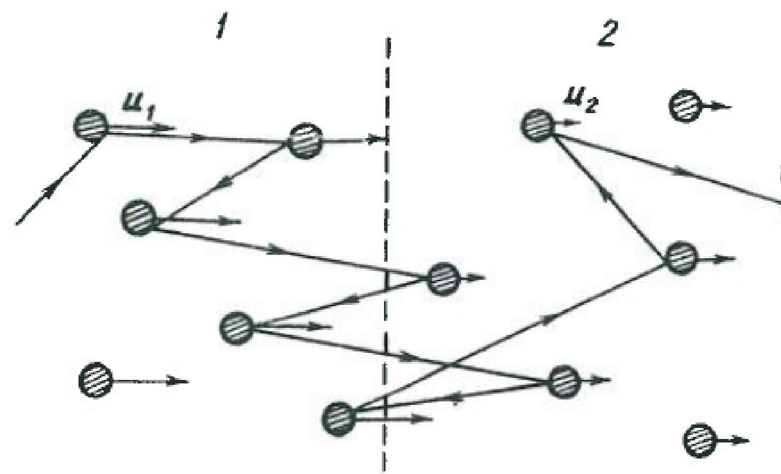
(Kowal et al. 2012)

Shock Acceleration

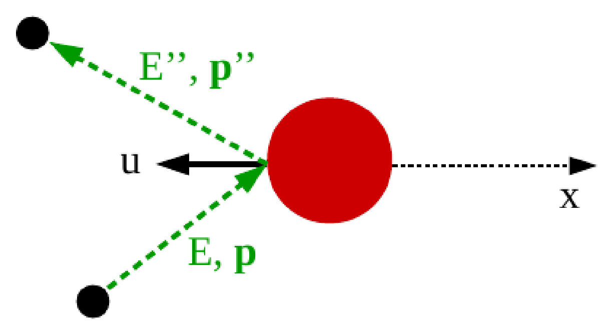

Cas A
(NASA/CXC/SAO)

(Caprioli & Spitkovsky 2014)





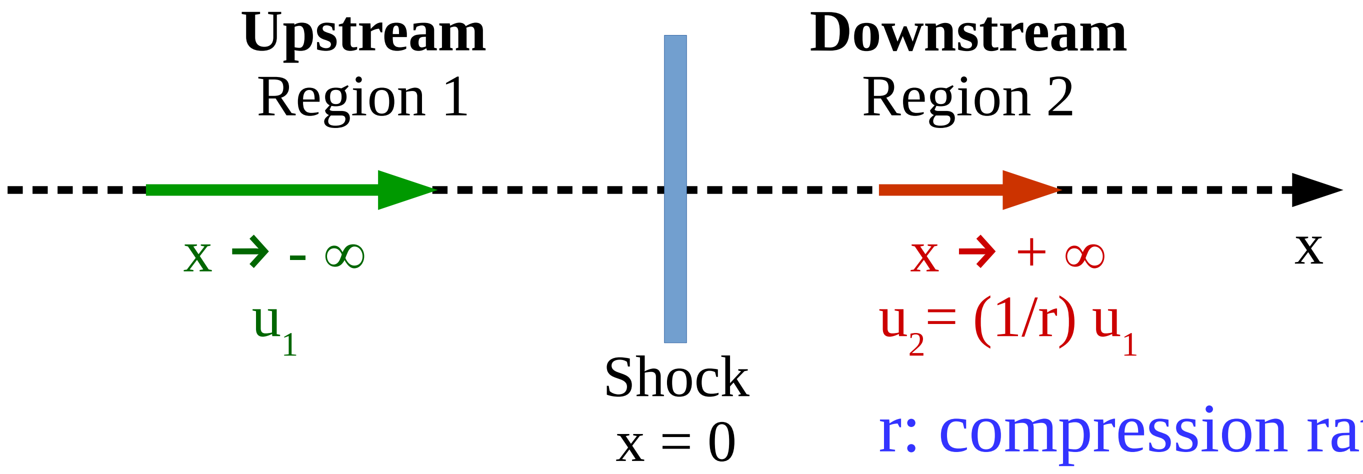
(Uzdenskii+ 1990)



SN shocks are the favored mechanism for acceleration CRs up to energies ~PeV

Converging flow at the shock ⇔ Fermi reflection across the shock front

Diffusive Shock Acceleration: 1st order Fermi acceleration



r: compression ratio

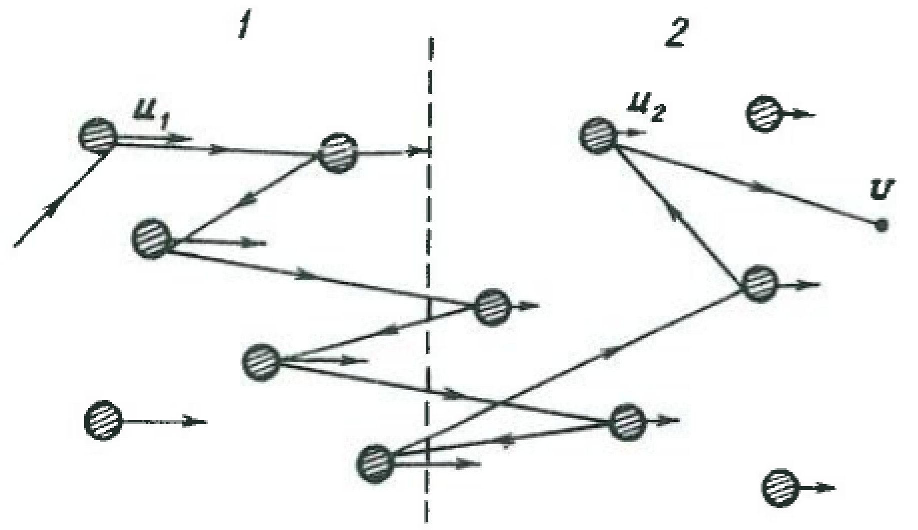
strong non-relarivistic shocks (monoatomic gas): r → 4

Transport equation in the diffusion approximation:

$$\frac{\partial f}{\partial t} + \nabla \cdot (\mathbf{u} f) + \nabla_{\mathbf{p}} \cdot \left(\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} f \right) = \nabla \cdot (D \nabla f)$$

$$f(\mathbf{x}, \mathbf{p}, t) = f(x, p)$$

Particle scattering:



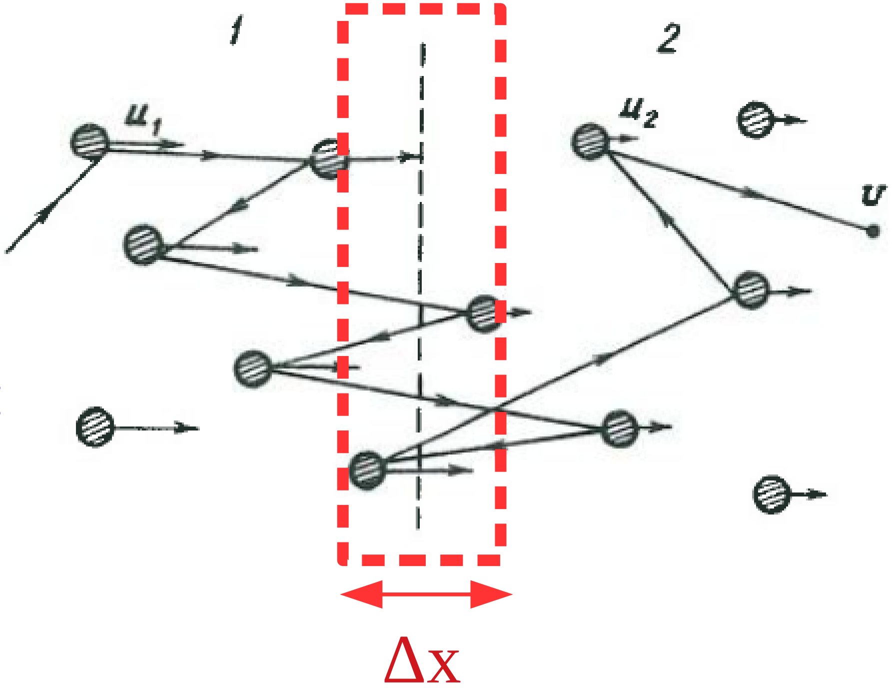
After a round trip (1 ⟶ 2 ⟶ 1), relative energy gain:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3}\beta = \frac{4}{3}\frac{(u_1 - u_2)}{c}$$

Change in energy proportional to the first order in the small parameter β

⇒ Fermi first order acceleration

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3}\beta = \frac{4}{3}\frac{(u_1 - u_2)}{c}$$



Relative gain in momentum (v ~ c):

$$\langle \Delta p \rangle = \frac{4}{3} p \frac{(u_1 - u_2)}{c}$$

Time to cross the distance Δx around the shock:

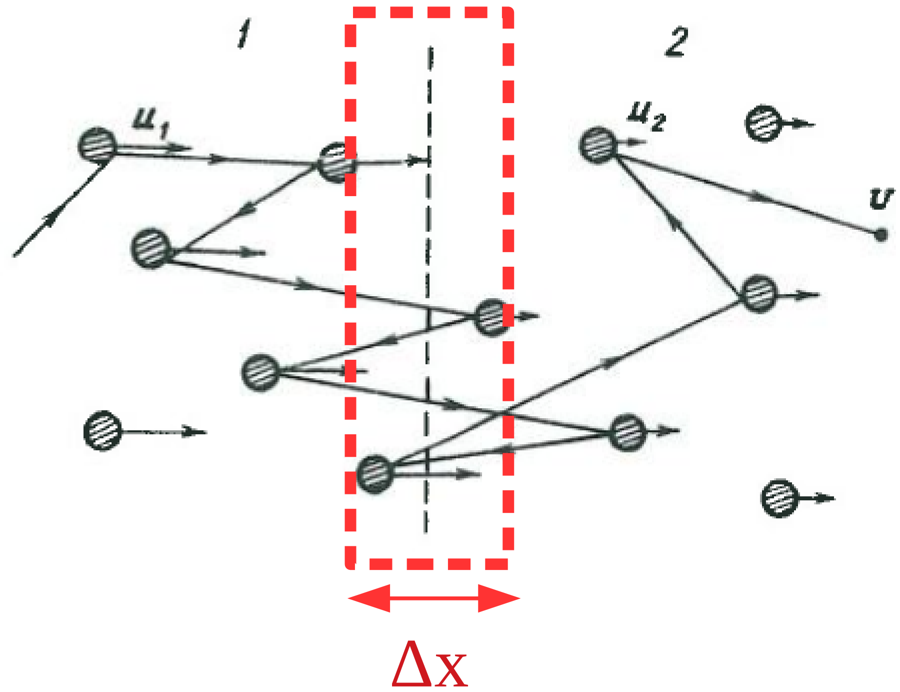
$$\tau_{\text{cross}} = \frac{\Delta x}{2} \frac{1}{\langle V_{1\to 2} \rangle} + \frac{\Delta x}{2} \frac{1}{\langle V_{2\to 1} \rangle}$$

Approx. the same average v_x speed in each side:

$$\tau_{\text{cross}} = \Delta x \frac{1}{\langle V_{1\to 2} \rangle}$$

$$\langle \Delta p \rangle = \frac{4}{3} p \frac{(u_1 - u_2)}{c}$$

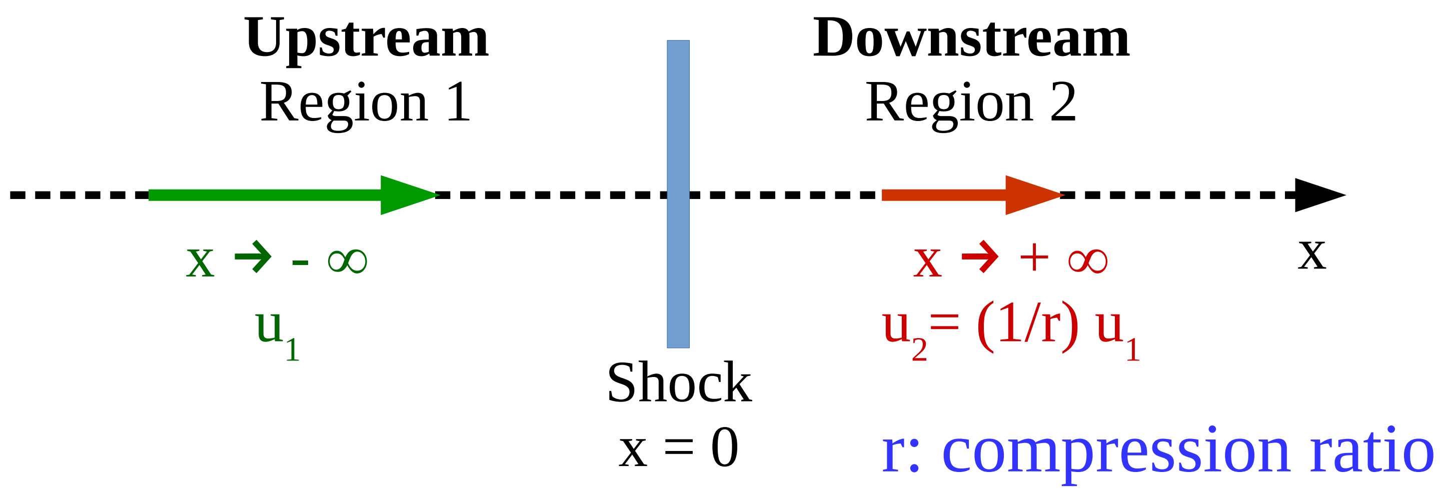
$$\tau_{\mathrm{cross}} = \Delta x \frac{1}{\langle V_{1\to 2} \rangle}$$



$$\tau_{\mathrm{cross}}^{-1} = \frac{1}{\Delta x} \langle V_{1\to 2} \rangle = \frac{1}{\Delta x} \int_0^1 \mathrm{d}\mu P_\mu v \mu = \frac{1}{4} \frac{v}{\Delta x}$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \langle \Delta p \rangle \tau_{\mathrm{cross}}^{-1} = \frac{4}{3} p \frac{(u_1 - u_2)}{c} \times \frac{1}{4} \frac{v}{\Delta x} = \frac{1}{3} p \left(-\frac{\Delta u}{\Delta x} \right)$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{1}{3} p \left(\frac{\partial u}{\partial x} \right)$$

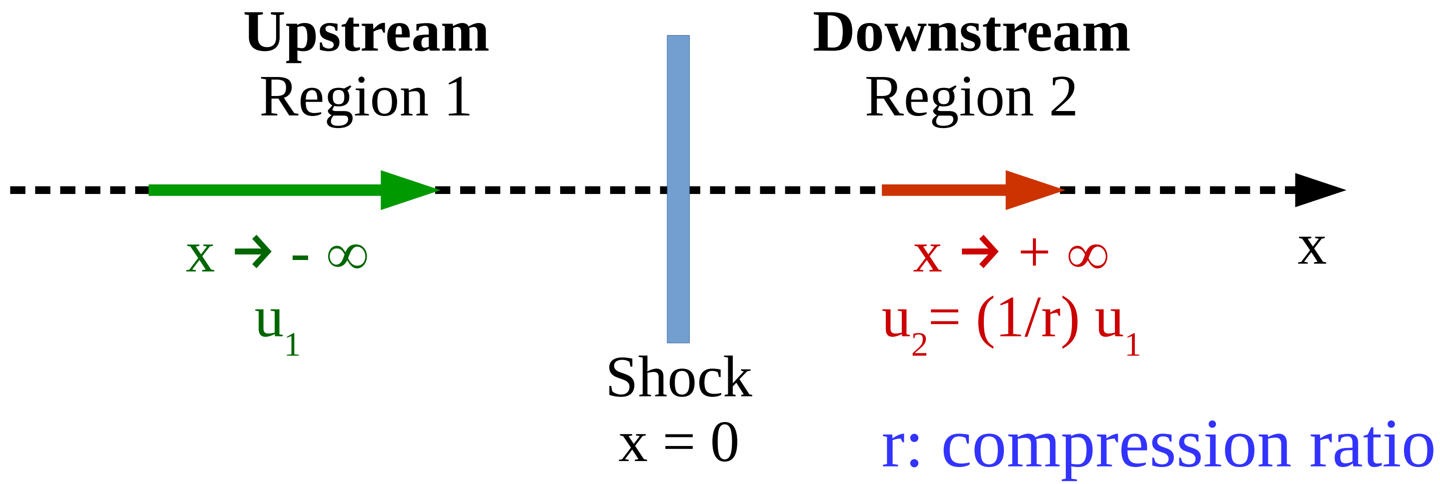


$$\frac{\partial f}{\partial t} + \nabla \cdot (\mathbf{u} f) + \nabla_{\mathbf{p}} \cdot \left(\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} f \right) = \nabla \cdot (D \nabla f)$$

$$f(\mathbf{x}, \mathbf{p}, t) = f(x, p)$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\left(\frac{p}{3}\right) \nabla \cdot \mathbf{u} = -\left(\frac{p}{3}\right) \frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial x} = (u_2 - u_1)\delta(x)$$



Stationary 1D transport equation:

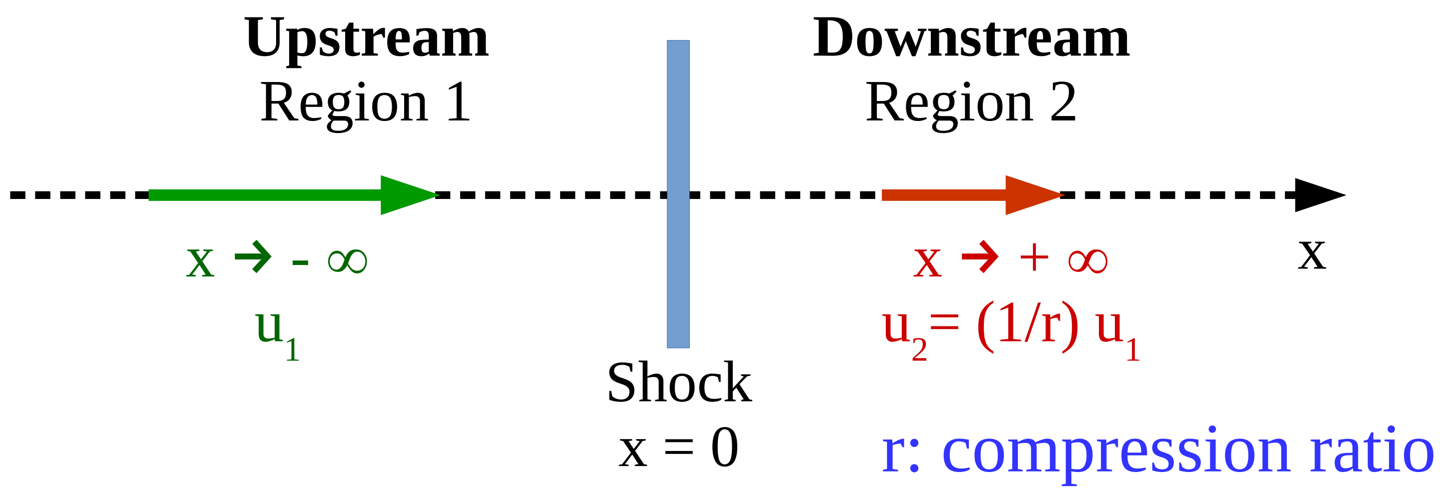
$$-\frac{\partial}{\partial x}\left(D\frac{\partial f}{\partial x} - uf\right) = \frac{1}{3}(u_2 - u_1)\delta(x)\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^3 f\right)$$

Continuous, no divergent solution across x=0:

$$f(p, x < 0) = C_1(p) + \{C_2(p) - C_1(p)\}\exp\left(\frac{u_1 x}{D}\right)$$

$$f(p, x > 0) = C_2(p)$$

(simplification: D constant)



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Integration around the shock:

$$\{u\}_0 f - \left\{D\frac{\partial f}{\partial x}\right\}_0 = \frac{(u_2 - u_1)}{3}\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^3 f\right) = 0$$

Continuous, no divergent solution across x=0:

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Integration around the shock:

$$\{u\}_0 f - \left\{D\frac{\partial f}{\partial x}\right\}_0 = \frac{(u_2-u_1)}{3}\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^3 f\right) = 0$$

… leads to the dependence on p:

$$p\frac{\mathrm{d}C_2}{\mathrm{d}p} + \frac{3u_1}{(u_2-u_1)}C_2 = \frac{3u_1}{(u_2-u_1)}C_1$$

$$p\frac{\mathrm{d}C_2}{\mathrm{d}p} + \frac{3u_1}{(u_2 - u_1)}C_2 = \frac{3u_1}{(u_2 - u_1)}C_1$$

Solving for C_2(p):

$$C_1(p) = C_0\delta(p - p_0)$$

$$C_2(p > p_0) = \Gamma\frac{C_0}{p_0}\left(\frac{p}{p_0}\right)^{-\Gamma}$$

$$\Gamma = \frac{3u_1}{(u_1 - u_2)} = \frac{3u_1/u_2}{(u_1/u_2 - 1)} = \frac{3r}{(r - 1)}$$

r: compression ratio

strong non-relarivistic shocks (monoatomic gas): r → 4

Back to energy, in the ultra-relativistic case ($E_0 >> mc^2$):

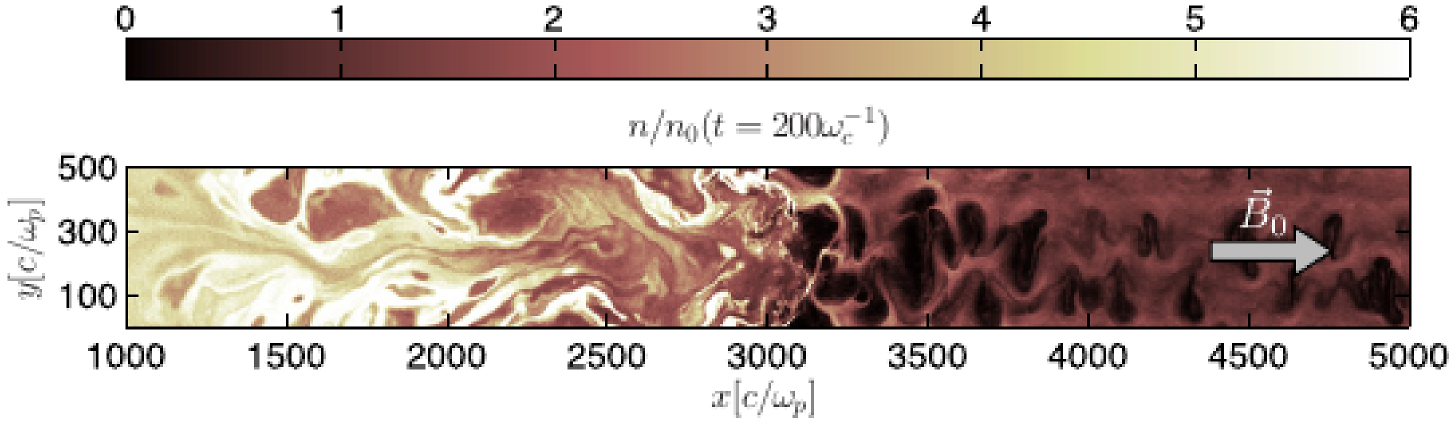
$$N(E) = C_0(\Gamma + 2)E_0^{-1} \left(\frac{E}{E_0}\right)^{-\Gamma} \quad \rightarrow \quad \boxed{N(E) \propto E^{-2}}$$

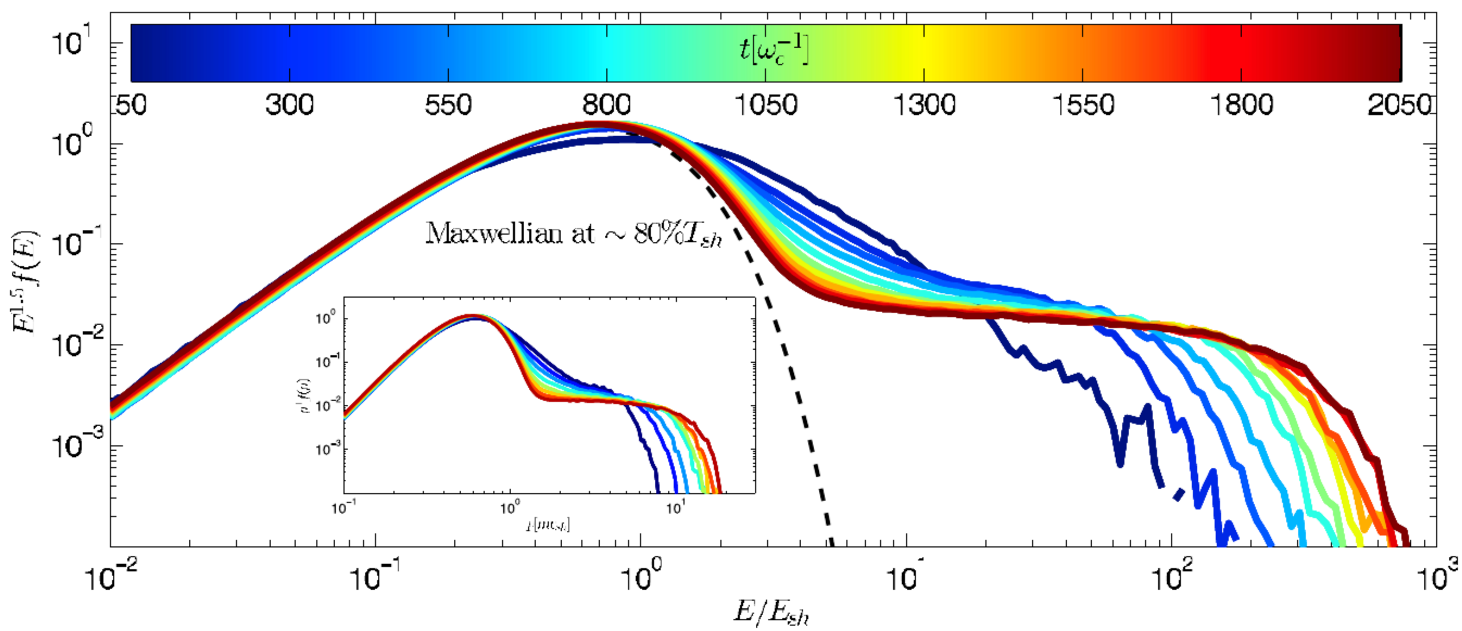
Non-relativistic case (in kinetic energy):

$$N(E_k) = C_0(\Gamma/2 + 1)E_{k0}^{-1} \left(\frac{E_k}{E_{k0}}\right)^{-(\Gamma+1)/2} \quad \rightarrow \quad \boxed{N(E_k) \propto E_k^{-1.5}}$$

- No dependence on the details of the scattering process
- E_{max}?

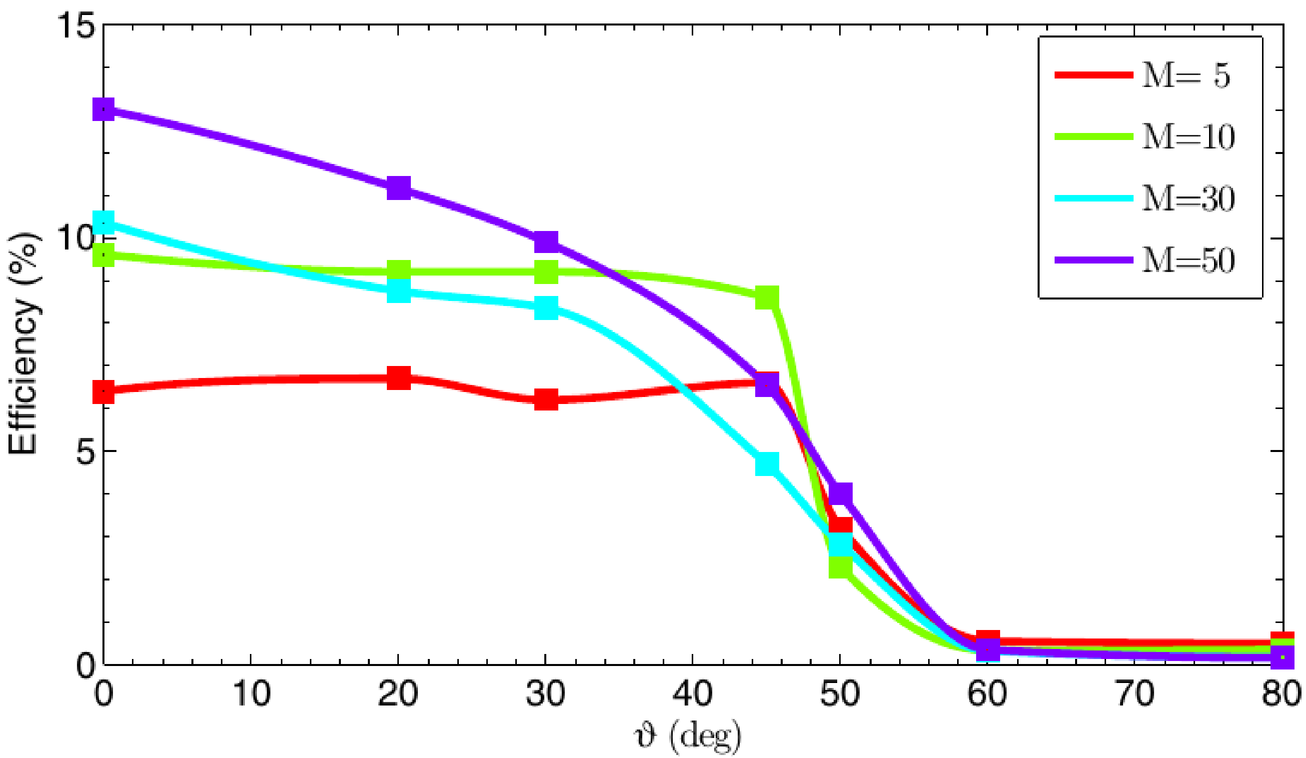
PIC simulation - non-relativistic shock, non-relativistic energies (Caprioli & Spitkovsky 2014)





PIC simulation - non-relativistic shock, non-relativistic energies (Caprioli & Spitkovsky 2014)

Efficiency for different magnetic field angles:



Non-linear theory of Diffusive Shock Acceleration

- PIC simulations show that up to 10-20% of the shock energy can be channeled to accelerated ⇒ particles particles dynamically important in the shock evolution

- To achieve ~PeV, amplification of the ambient magnetic field by a factor of dozens is needed in the pre-shock (to increase confinement):
  - Amplification by non-resonant streaming instabilities?
  - Amplification by a CR driven dynamo in the precursor?

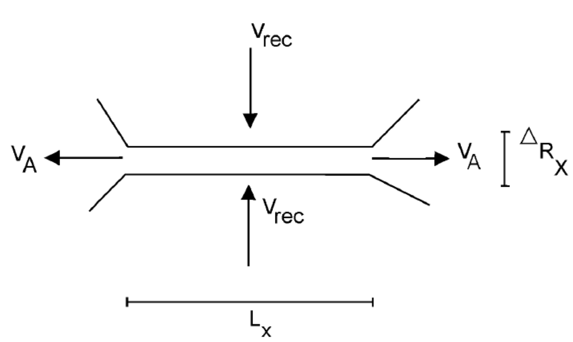
- Nature and evolution of the scatters: self-generated waves, role of pre-existing turbulence and inhomogeneities, CR induced turbulence, damping processes, role of reconnection in the turbulence

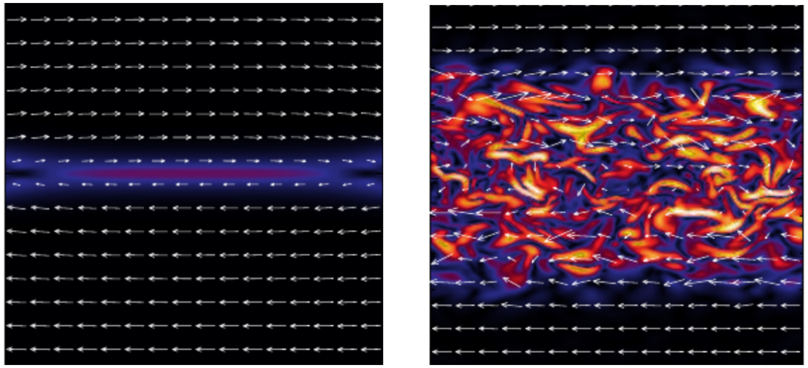
- Needs self-consistent approach, covering plasma phenomena in several scales ⇒ computationally challenging

See e.g. Malkov & Drury 2001; Schure et al. 2012; Blasi 2013; Bykov et al. 2013, 2018; Caprioly & Spitskovsky 2014a,b,c; del Valle at al. 2016; Inoue et al. 2021; Caprioli 2023

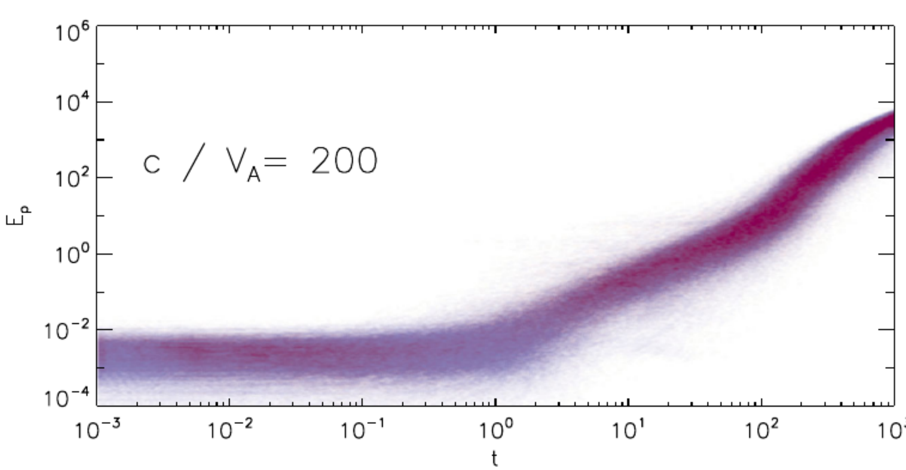
Diffusive reconnection acceleration?

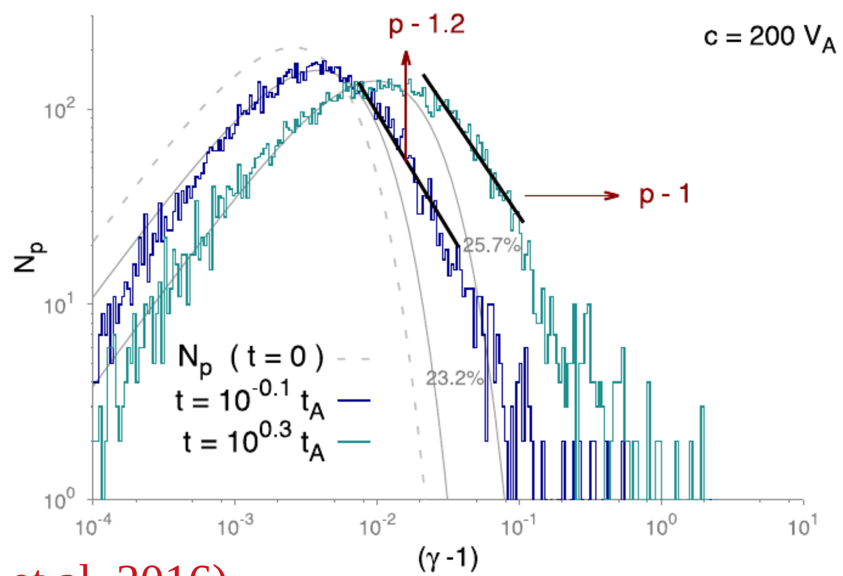
- First order diffusive process aroung the converging flows in the turbulent reconnection process (de Gouveia Dal Pino & Lazarian 2005)





(Kowal et al. 2012)





(del Valle et al. 2016)

To remember

- Efficient particle acceleration during magnetic reconnection occurs mainly due to Fermi reflection and the betatron effect. Direct acceleration on the current sheet is secondary.
- Magnetic reconnection is thought be present in a large number of astrophysical sources and is likely turbulent. Still the subject of intense research.
- Astrophysical systems are almost always turbulent. The 2nd order Fermi diffusive process due to particle scattering by MHD turbulent waves can produce non-thermal particles with a power-law in energy.
- First order Fermi diffusive acceleration occurs in shocks (DSA) and produces non-thermal particles. Basic theory predicts a power-law index -2, without dependence on the details of the scattering process.
- The complexities of the DSA scenario involve the maximum energy achieved by the particles, the production of waves, the magnetic amplification and the modifications of the shock evolution. The approach to these problems require self-consistent approaches.
- Numerical simulations using PIC (plasma + self-consistent fields) or Test-Particles have been fundamental for testing theories and exploring systems with complex configurations.