Lectures on quantum gravity phenomenology

Part 1: Theoretical models

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Short Course on: Theory and data analysis of Astroparticles **IFSC-USP**

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21.02.2024

from Bronstein's work in 1936



Matvei Bronstein, "Republication of: Quantum theory of weak gravitational fields", Gen Relativ Gravit (2012) 44:267–283

The quantization of the spacetime metric leads to an uncertainty in the determination of the trajectory probed by a test particle when gravity is "turned on".

something else

Matvei Bronstein - One of the first researchers of quantum gravity in 1936

Geometric uncertainty

A historical description of quantum gravity can be seen in many books on the subject, for example from Rovelli's book "REALITY IS NOT WHAT IT SEEMS", which describes the birth of quantum gravity

Spacetime would no longer be Riemannian, but would need to be

$$\Delta[00,1] \gtrsim \frac{h^{2/3} G^{1/3}}{c^{1/3} \rho^{1/3} V^{2/3} T}.$$



This makes one suspect that quantum gravity leads to fundamental modifications in the geometrical nature of spacetime

One would also suspect that quantum mechanics postulates need to be modified.

The uncertainty in the connection is related to an uncertainty in the momentum of test particles.

This uncertainty sums up to the known Heisenberg uncertainty



Atoms of spacetime

 $\Delta p_x \approx \frac{h}{\Delta x} + G\rho^2 V \Delta x \Delta t.$





Quantum Gravity corrections to known physics at an intermediate eve

Riemannian Geometry + corrections

"Standard" Quantum Mechanics + corrections



Examples:

There exist many other examples of approaches to quantum gravity that point to **departures of the Riemannian description of** spacetime and its symmetries or the **standard quantum** mechanics/quantum field theory

• Loop Quantum Gravity [Amelino-Camelia, da Silva, Ronco, Cesarini, Lecian, PRD (2017)]

• Horava-Lifshitz gravity [P. Horava, PRL (2009)]

 Causal Dynamical Triangulation [Ambjorn, Jurkiewicz, Loll, PRL (2005)]

• Non-critical Lioville string theory [Amelino-Camelia, Ellis, Mavromatos, Nanopoulos, IJMPA (1997)]

QG correction to Schrödinger equation [Kiefer, Singh, PRD (1991)]

3D Quantum Gravity [Freidel, Levine, PRL (2006)]





Example: Hypersurface Deformation Algebra from Loop Quantum Gravity



It's possible to write the equations of general relativity in terms of the algebra of generators of time evolution and space diffeomorphism

Effectively, one approaches Loop Quantum Gravity corrections of GR by deforming these generators

A linearization of this deformed algebra leads to a modification of Poincaré algebra with mass shell

$$m^2 \approx E^2 - P^2 - \lambda^2 P^4$$

Brahma, Ronco, Amelino-Camelia, Marciano, PRD (2017)







What is the correct approach to quantum gravity?



Can we test these theories of QG?

What is a test of the quantum nature of spacetime?





Quantum gravitation phenomenology is a **bottom-up** proposal, in which the varied possibilities of **modifications of** basic and well-established equations of relativity are parameterized through corrections inspired by expectations of what semi-classical limits would be like or lessons from theories that quantize the gravitational field (or the geometry of space-time).









Quantum gravity needs experiments!

Quantum gravity is a long-lived research area that has achieved enough maturity to be studied at a **phenomenological** level

• In the past 15 years, several effects predicted by approaches to quantum gravity have been constrained with Planck scale sensitivity

What is Planck scale sensitivity?

• <u>Planck scale sensitivity</u>:

Ability of an experimental setup or measurement technique to detect variations, parametrized by Planckian units, in the quantity or phenomenon being measured

length or the inverse Planck energy and the energy scale of a particle $\ell_P E$. These corrections would be too tiny to be detected as $\ell_P^{-1} \sim 10^{28} \,\mathrm{eV}$

have the most optimistic corrections in particles interactions of the order 10^{-7}

• Naively, one could expect that such variations would be simply given by powers of the product of Planck

• For example, considering that the most energetic cosmic rays (UHECR) reach $10^{20} - 10^{21} \text{ eV}$, we would



present **amplifiers** that would allow measurements with Planck scale sensitivity

• Such amplifiers can appear both in the relativistic **ultraviolet** regime, from tens of GeV till beyond the PeV scale, or in the non-relativistic infrared regime.

• Amplifiers can come in **many forms** when coupled to the Planck length

Although this is correct for many observables, for a long time it was overlooked that phenomena could



Brownian motion





• small contributions add up to a measurable effect

Example: In-vacuo dispersion

- Around the beginning of the 21st century, technological advances (accuracy of experiments) allowed a different approach to be proposed
- If spacetime is discretized, one can **expect small** corrections in the kinematics of particles propagating in this background







quantum spacetime

$$m^{2} = E^{2} - p^{2} + \frac{1}{E_{Pl}} \left(\alpha E^{3} + \beta E^{2} p \dots \right) + \frac{1}{E_{Pl}^{2}} \left(\gamma E^{4} + \lambda E^{2} p^{2} \dots \right) + \dots$$

Modified Dispersion Relations (MDR)

Inspired by

- LQG [Amelino-Camelia, da Silva, Ronco, Cesarini, Lecian, PRD (2017)]
- Horava-Lifshitz gravity [P. Horava, PRL (2009)]
- CDT [Ambjorn, Jurkiewicz, Loll, PRL (2005)]
- Non-critical Lioville string theory [Amelino-Camelia, Ellis, Mavromatos, Nanopoulos, IJMPA (1997)]

• Effectively, it is possible to capture modifications of the kinematics of particles when they travel through a

This idea gives rise to a wide phenomenology based on the modified trajectories that particles follow and modifications in processes involving fundamental particles in comparison to special relativity, which can be tested using cosmic messengers



Nontrivial Minkowski limit



So, one would find a non-trivial local Minkowskian limit

Reminiscent of the Planck scale would remain at the Minkowskian limit

E_{Planck} remains finite

Modified dispersion relation is not invariant under the action of the Lorentz group

To violate Lorentz simmetry

• Different inertial frames measure different **MDRs**









To deform or to extend the Lorentz/Poincaré symmetry

• Energy and momentum, in each frame, are related by a deformation of the usual Poincaré transformation that preserves the MDR.



- MDR for elementary particles
- Deformed Lorentz transformation

• The invariance of the new dispersion relation under the new frame transformation is assured if

 $\Lambda[\mathscr{H}(E,p)] = \mathscr{H}(E',p')$





It is necessary to change the composition law non-linearly

$$\begin{cases} p_{\mu} \bigoplus q_{\mu} = p_{\mu} + q_{\mu} + \ell f_{\mu}(p,q) + \dots \\ \Lambda_{\ell}(p \bigoplus q) = \Lambda_{\ell}(p) \bigoplus \Lambda_{\ell}(q) \end{cases}$$

• Giovanni Amelino-Camelia, IJMPD (2002) João Magueijo, Lee Smolin, PRL (2002)





IN THE CONTEXT OF MODIFIED DISPERSION RELATIONS

LORENTZ INVARIANCE VIOLATION

MDR
$$E^2 = m^2 + p^2 + s\eta^{(n)} \frac{p^{n+2}}{E_{Pl}^n}$$

- $s = 1 \Rightarrow$ Superluminal propagation
- $s = -1 \Rightarrow$ Subluminal propagation

 $\eta^{(n)}$ is the dimensionless parameter to be constrained

No symmetries

$$p_{\mu} \oplus q_{\mu} = p_{\mu} + q_{\mu}$$

 $\Lambda_{SR}(p \oplus q) = \Lambda_{SR}(p) \oplus \Lambda_{SR}(q)$



Modified Symmetries and Composition Law

 $p_{\mu} \oplus q_{\mu} = p_{\mu} + q_{\mu} + \text{corrections}$

 $\Lambda_{\ell}(p \oplus q) = \Lambda_{\ell}(p) \oplus \Lambda_{\ell}(q)$



LIV (Lorentz Invariance Violation) has been explored in Humberto's lecture. It requires the introduction of new terms in the action of interactions with LIV contributions. But one proceeds using the standard Riemannian (Minkowski) geometrical language

Ex.: Standard Model Extension, by Kostelecký et al.

Lorentz-Violating Extension of the Standard Model

D. Colladay and V. Alan Kostelecký Physics Department, Indiana University, Bloomington, IN 47405, U.S.A. (preprint IUHET 359 (1997); accepted for publication in Phys. Rev. D)

Some entry points for SME:

Colladay, Kostelecky, PRD 1998 Kostelecky, Lane, PRD 1999 Mattingly, Liv. Rev. Rel. 2005 Kostelecky, Russell, Rev. Mod. Phys. 2011

For modeling a deformation of Lorentz symmetry, one needs to consider alternative mathematical frameworks

NONCOMMUTATIVE GEOMETRY

but deformed, this needs to be considered when modeling these effects.

Noncommutative geometry (*k*-Minkowski spacetime)

Lukierski, Ruegg, Nowicki, Tolstoy, PLB 1991

Presents deformed generators of translations, boosts and rotations (*k*-Poincaré algebra)

$$\begin{aligned} & [J_a, J_b] = \epsilon_{abc} J_c \,, & [J_a, P_b] = \epsilon_{abc} P_c \,, & [J_a, K_b] = \epsilon_{abc} K_c \,, \\ & [K_a, P_0] = P_a \,, & [K_a, K_b] = -\epsilon_{abc} J_c \,, \\ & [P_0, P_a] = 0 \,, & [P_a, P_b] = 0 \,, & [P_0, J_a] = 0 \,, \end{aligned}$$

$$[K_a, P_b] = \delta_{ab} \left(\frac{\kappa}{2} \left(1 - e^{-2P_0/\kappa} \right) + \frac{1}{2\kappa} \mathbf{P}^2 \right) - \frac{1}{\kappa} P_a P_b \,.$$

Mass Casimir or MDR

$$C_{\kappa} = 4\kappa^2 \sinh^2(P_0/2\kappa) - e^{P_0/\kappa} \mathbf{P}^2,$$

There are many mathematical languages to describe DSR models. Since the underlying symmetry is not broke

$$[x_m, t] = \frac{i}{\kappa} x_m , \quad [x_m, x_l] = 0$$

Coproduct of the algebra gives the modified composition law

$$P_0 \bigoplus Q_0 = P_0 + Q_0$$
$$P_1 \bigoplus Q_1 = P_1 + e^{-P_0/k}Q_1$$

Poincaré algebra ($\kappa \to \infty$ **)**

$$[K_a, P_b] = \delta_{ab} P_0 \,,$$

$$\mathcal{C}=P_0^2-\mathbf{P}^2\,,$$

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CURVED MOMENTUM SPACE

Curved momentum space

interactions is related to a curvature of momentum space (Born Reciprocity Principle from 1938).

By M. BORN

Quantized Space-Time

HARTLAND S. SNYDER Department of Physics, Northwestern University, Evanston, Illinois (Received May 13, 1946)

Apparently independently discovered by Snyder and followed by Russian physicists from the 50's to the 80's Amelino-Camelia, Freidel, Kowalski-Glikman, Smolin, PRD 2011

In Special Relativity, momentum space is flat. So, maybe the nontrivial structure of momentum space and

A suggestion for unifying quantum theory and relativity

(Communicated by E. T. Whittaker, F.R.S.—Received 5 January 1938)



QG Phase space diagram

$$\Gamma^{\rm DSR} = T^* \mathscr{P}_{\checkmark}$$

P(nomentur SPACE) $\Gamma^{SR}_{X} = \mathscr{M} \times \mathscr{P}$ $\Gamma^{\rm GR} = T^* \mathcal{M}$ $\Gamma^{QG} = ?$

The deformed Poincaré group is actually the group of symmetries in a maximally symmetric momentum space

De Sitter momentum space gives *k*-Poincaré symmetries

Gubitosi, Mercati, CQG (2013)



Mass shell is defined from geodesic distance in momentum space

Spacetime is given by the covectors in this momentum space

Anti-de Sitter momentum space gives the momentum space of 2+1 QG

IPL, Amelino-Camelina, Palmisano, arXiv:2024.xxxx

The curvature is the inverse of the Planck energy $R = E_{Pl}^{-2}$, such that when $E_{Pl} \to \infty$, we recover SR











FINSLER GEOMETRY

Relativity was mostly clarified after Minkowski introduced the geometric description of spacetime

- Relativity principle
 Isometries are defined in a riemannian
 spacetime, since the dispersion relation is the
 norm of the 4-momentum
- Equivalence principle (GR)
 Free particles follow riemanniana
 geodesics

• Clock postulate

Observers measure their proper time by the arc length function.

DSR Formalism

Relativity principle Transformations that preserve the MDR *H*

• Trajectories

Defined by $dx/dt = \partial E/dp|_{\mathscr{H}=m^2}$

• Proper time

There exists a geometric formalism that can be to Doubly Special Relativity (DSR) what riemannian geometry is to Special Relativity (SR)?

Special relativity





Minkowski

Riemann











• The action of a free particle is of the form

$$S[x, p, \lambda]_{H} = \int d\mu (\dot{x}^{\mu} p_{\mu} - \lambda f(\mathcal{H}(x, p), m))$$

- 1) Variation with respect to λ enforces the dispersion relation.
- allow to eliminate the momenta p_{μ} of the action
- 3) Using $p_{\alpha}(x, \dot{x}, \lambda)$ on the MDR, one can find $\lambda(x, \dot{x})$ (this can only be done for massive particles).
- 4) Finally the equivalent action is obtained as $S[x] = S[x, p(x, \dot{x}, \lambda(x, \dot{x})), \lambda(x, \dot{x})]_{H}$.

Algorith and general Finsler function

[Girelli, Liberati, Sindoni, PRD (2007)] **[IPL**, Christian Pfeifer, PRD (2021)]

 $f = 0 \Leftrightarrow \mathscr{H}(x, p) = m^2$

2) Variation with respect to p_{μ} takes us to an equation $\dot{x}^a = \dot{x}^a(p,\lambda)$, which must be inverted to give $p_{\mu}(x,\dot{x},\lambda)$ and



• Approximately, a modified hamiltonian has the form

 $H(x,p) = g(p,p) + \epsilon h(x,p)$ Perturbation parameter

• It gives the following action in spacetime



Constructing Finsler geometry

where
$$h(x,p) = h^{\mu_1 \mu_2 \dots \mu_n}(x) p_{\mu_1} p_{\mu_2} \dots p_{\mu_n}$$

$$S[x] = m \int F(x, \dot{x}) d\mu$$

$$F(x, \dot{x}) = \sqrt{g(\dot{x}, \dot{x})} - \epsilon m^{n-2} \frac{h_{\mu_1 \mu_2 \dots \mu_n}(x) \dot{x}^{\mu_1} \dot{x}^{\mu_2} \dots \dot{x}^{\mu_n}}{2g(\dot{x}, \dot{x})^{\frac{n-1}{2}}}$$

• The connection with Finsler geometry is realized by the identification of the arc length functional, s[x], for massive observers, from which the Finsler metric $g_{\mu\nu} = \partial^2 (F^2/2) / \partial \dot{x}^{\mu} \partial \dot{x}^{\nu}$ e $\mathcal{H} = g^{\mu\nu} p_{\mu} p_{\nu}$ can be found

$$\doteq m^{-1}S[x]$$





Deformed trajectories are **geodesics** that extremize the Finsler arc-length

$$\dot{x}^{\mu} + \Gamma(x, \dot{x})^{\alpha}_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} = \frac{\dot{F}}{F} \dot{x}^{\mu}$$

Coincide with the trajectories found from $dx/dt = \partial E/dp|_{\mathscr{H}=m^2}$

$$\begin{cases} x(t) = \frac{p}{\sqrt{m^2 + p^2}} t + \ell_{QG} p t + \dots \\ x(t) = t + \ell_{QG} p t + \dots \end{cases}$$
 Time delay

[Amelino-Camelia, Barcaroli, Gubitosi, Liberati, Loret, PRD (2014)], [IPL, Niccolò Loret, Francisco Nettel, PRD (2017)], [Zhu, Ma, EPJC (2023)],

 $\Gamma^{\alpha}_{\mu\nu}(x,\dot{x}) = \frac{1}{2}g^{\alpha\beta}(x,\dot{x})\left(g_{\beta\mu,\nu}(x,\dot{x}) + g_{\beta\nu,\mu}(x,\dot{x}) - g_{\mu\nu,\beta}(x,\dot{x})\right)$





• Symmetries are deformed from Killing vectors

$$g_{(\mu\rho}\partial_{\nu)}\xi^{\rho} + \frac{\partial g_{\mu\nu}}{\partial x^{\rho}} + \frac{g_{\mu\nu}}{\partial \dot{x}^{\rho}} \frac{\partial \xi^{\rho}}{\partial x^{\sigma}} \dot{x}^{\sigma} = 0$$

$$\begin{cases} \tilde{E} \approx E + vp \\ \tilde{p} \approx p + vE - \ell_{QG} v \left(E^2 + \frac{p^2}{2} \right) \end{cases}$$

[Amelino-Camelia, Barcaroli, Gubitosi, Liberati, Loret, PRD (2014)], [IPL, Christian Pfeifer, Pedro H. Morais, Rafael Alves Batista, Valdir B. Bezerra, JHEP (2022)] [Pedro H. Morais, IPL, Christian Pfeifer, Rafael Alves Batista, Valdir B. Bezerra, PLB (2023)]

Symmetries that preserver the mass shell $\mathscr{H} = g^{\mu\nu}(x, \dot{x})p_{\mu}p_{\nu}$



HAMILTON GEOMETRY

MDR
$$m^2 = H(E, p)$$

Metric $g^{\mu\nu} = \frac{\partial^2 H}{\partial p_{\mu} \partial p_{\nu}}$

- This metric is not invariant under reparametrizations, therefore it cannot define an arc-length
- I'm not sure if we can call this approach a geometry, since it can't be used to measure distances
- Nevertheless, the trajectories are found from the Hamilton equations.
- Symmetries can be defined as well from a Killing equation
- There is a canonical connection as well

Barcaroli, Brunkhorst, Gubitosi, Loret, Pfeifer, PRD (2015)

It's possible to define connections, curvature, etc. All the known geometric quantities of riemannian geometry and beyond

Pedagogical book on Finsler and Hamilton geometries



Quantum Configuration and Phase Spaces: Finsler and Hamilton Geometries

Saulo Albuquerque,^{1,*} Valdir B. Bezerra,^{1,†} Iarley P. Lobo,^{2,3,‡} Gabriel Macedo,^{1,§} Pedro H. Morais,^{1,¶} Ernesto Rodrigues,^{1,**} Luis C. N. Santos,^{1,††} and Gislaine Varão^{1,‡‡}

¹Physics Department, Federal University of Paraíba, Caixa Postal 5008, 58059-900, João Pessoa, PB, Brazil. ²Department of Chemistry and Physics, Federal University of Paraíba, Rodovia BR 079 - Km 12, 58397-000 Areia-PB, Brazil. ³Physics Department, Federal University of Lavras, Caixa Postal 3037, 37200-000 Lavras-MG, Brazil. (Dated: January 24, 2023)

Our review paper on applications to quantum gravity phenomenology

arXiv:2301.09448





SOME IMMEDIATE EFFECTS



Considering a MDR





Time delay due to modified trajectories



This effect is present both in LIV and DSR

Modified Trajectories

R





LIV

Processes that are forbidden in SR are allowed in LIV

The energy threshold of processes of SR are strongly modified in LIV

Ex.: Photon decay is allowed in LIV $\gamma \rightarrow e^+e^-$

$$\cos(\theta) \simeq \frac{E_{+}(E_{\gamma}-E_{+})+m_{e}^{2}-\eta E_{\gamma}E_{+}(E_{\gamma}-E_{+})/E_{p}}{E_{+}(E_{\gamma}-E_{+})}$$

Opening angle between e^+ and e^- can be < 1

Properties of interactions depend on the dispersion relation and on the conservation law

DSR

Processes that are forbidden in SR are still forbidden in DSR

The energy threshold of processes of SR are mildly modified in LIV

$$E_{\gamma} \simeq E_+ + E_- - \eta \vec{p}_+ \cdot \vec{p}_- ,$$

$$\vec{p}_{\gamma} \simeq \vec{p}_{+} + \vec{p}_{-} - \eta E_{+}\vec{p}_{-} - \eta E_{-}\vec{p}_{+}$$
.

$$\cos(\theta) \simeq \frac{2E_{+}(E_{\gamma}-E_{+})+\eta E_{\gamma}E_{+}(E_{\gamma}-E_{+})+2m_{e}^{2}}{2E_{+}(E_{\gamma}-E_{+})+\eta E_{\gamma}E_{+}(E_{\gamma}-E_{+})}$$

Always larger than 1

Amelino-Camelia, Liv. Rev. Rel. (2013)





Not only trajectories are modified, but the emission and detection events are distant and differ by a translation, and depending on the emission event, the boost cannot be disregarded

This modifies the prediction for time delays

Modified rotation

Usually rotations are not deformed due to the isotropic nature of the dispersion relation



Beyond Modified Dispersion Relations

Quantum gravity may introduce an extra degree of uncertainty in spacetime, beyond the usual Heisenberg principle

$$[\hat{x}^{i}, \hat{p}_{j}] = i \left[1 + \alpha \frac{\hat{p}}{E_{Pl}} + \beta \frac{\hat{p}^{2}}{E_{Pl}^{2}} + \dots \right] \delta_{j}^{i}$$

It's possible to map this formalism into one that present equation

$$[\hat{x}^i, \hat{k}_j] = i\delta^i_j, \qquad \qquad \hat{H} = \frac{\hat{k}^2}{2M} + V(\hat{x}) + \delta H(\hat{k}).$$

This leads to several effects like violation of the equivalence principle, decoherence by a quantum spacetime, shift in energy levels, that are being constrained nowadays with Planck scale sensitivity.

$$\hat{H} = \frac{\hat{p}^2}{2M} + V(\hat{x}),$$



Wagner, Varão, Lobo, Bezerra, PRD (2023)

Amelino-Camelia, Laemmerzahl, Mercati, Tino, PRL (2009)







Violation of Pauli Exclusion Principle (spin statistics) **Piscicchia et al., PRL (2022)**

CPT violation and decoherence

Spacetime fuzziness

Giovanni Amelino-Camelia, "Quantum Spacetime Phenomenology", Living Reviews in Relativity 16 (2013) 5 arXiv:0806.0339

Please refer to

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Andrea Addazzi et al., "Quantum gravity phenomenology at the dawn of the multimessenger era -- A review", Progress in Particle and Nuclear Physics 125 (2022) 103948 arXiv:2111.05659

Rafael Alves Batista et al., "White Paper and Roadmap for Quantum Gravity Phenomenology in the Multi-Messenger Era", arXiv:2312.00409

Mavromatos, Lect. Not. Phys. (2005)

Vasileiou, Granot, Piran, Amelino-Camelia, Nature Physics (2015) **Petruzziello, Illuminati, Nature Communications (2021)**



As a continuation of the activities of a recent COST Action, called "Quantum Gravity Phenomenology in the Multimessenger Approach" (from which Humberto and myself are part of).

The members created a Network, called QGMM, intended to bring closer experimentalists and theoreticians to push forward this area.



etc...

https://sites.google.com/view/qgmm/newsletter/sign-up?authuser=0



We have a newsletter to be informed about relevant information for our community, like calls for positions, conferences, schools,

Thank you! Obrigado!