Lectures on quantum gravity phenomenology

Part 2: LIV vs DSR from observables

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Short Course on: Theory and data analysis of Astroparticles

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We recall the main differences between LIV and DSR

LORENTZ INVARIANCE VIOLATION

MDR $$E^2 = m^2 + p^2 + s\eta^{(n)}\frac{p^{n+2}}{E_{Pl}^n}$$

$s = 1 \Rightarrow$ Superluminal propagation

$s = -1 \Rightarrow$ Subluminal propagation

$\eta^{(n)}$ is the dimensionless parameter to be constrained

SR Composition law $$p_\mu \oplus q_\mu = p_\mu + q_\mu$$

$$\Lambda_{SR}(p \oplus q) = \Lambda_{SR}(p) \oplus \Lambda_{SR}(q)$$

DEFORMED SPECIAL RELATIVITY

OR

LORENTZ INVARIANCE DEFORMATION

MDR $$E^2 = m^2 + p^2 + s\eta^{(n)}\frac{p^{n+2}}{E_{Pl}^n}$$

Modified Symmetries and Composition Law $$p_\mu \oplus q_\mu = p_\mu + q_\mu + \text{corrections}$$

$$\Lambda_\ell(p \oplus q) = \Lambda_\ell(p) \oplus \Lambda_\ell(q)$$

These may seem very simple modifications, and you may wonder that they are not that important after all.

This is a wrong assumption, otherwise I wouldn't be here

In the following, I'm going to give you some examples of astrophysical observables and discuss how that are affected by these different assumptions.

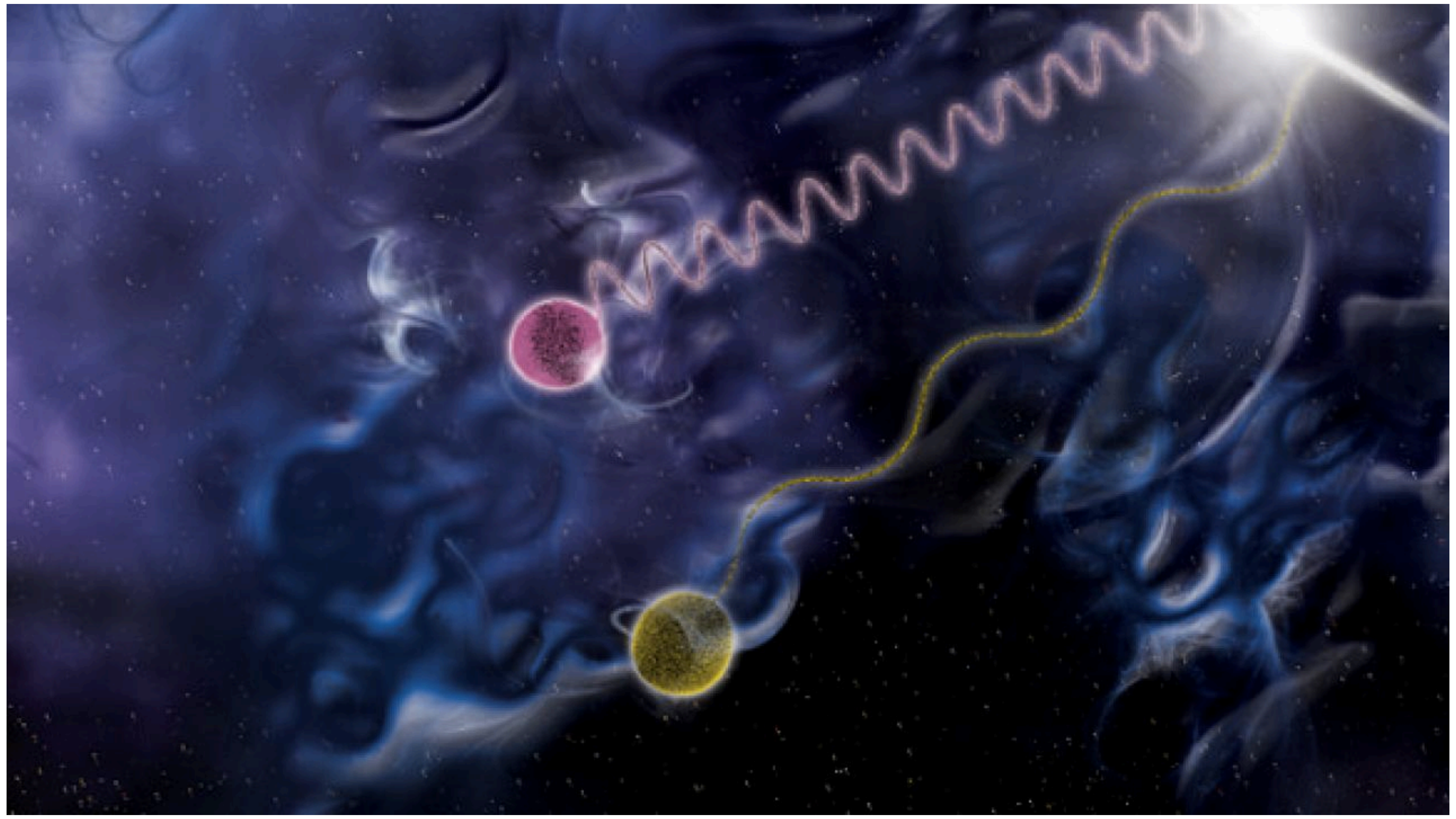
We're discussing **time delays**, **energy thresholds**, **propagation effects**, **time dilation**

Time delays

Example: In-vacuo dispersion

- Around the beginning of the 21st century, technological advances (accuracy of experiments) allowed a different approach to be proposed

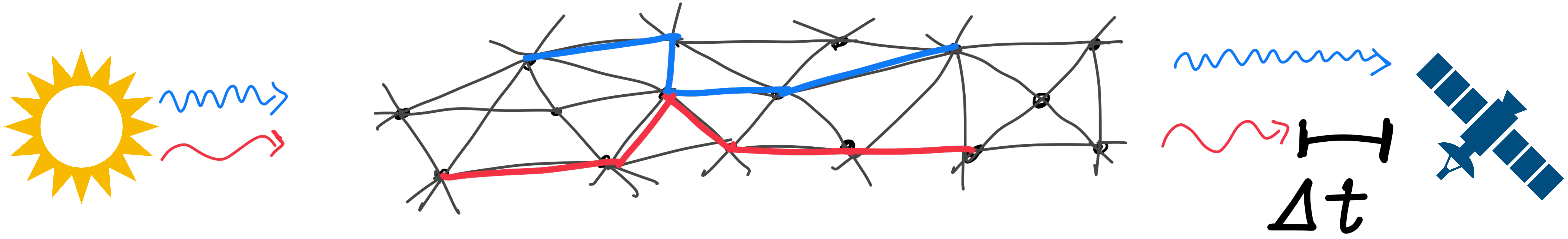
- If spacetime is discretized, one can **expect small corrections** in the kinematics of particles **propagating** in this background



QG phenomenology in the UV - in-vacuo dispersion

- A common intuition of QG is that a quantum spacetime would behave like a medium where particles propagate, whose irregularities would be of Planckian size (or of whatever the QG ruling scale is)

- Time-delays from GRBs, AGNs, Pulsars



Propagation distance is an amplifier to tiny local effects

$$E^2 - p^2 - \xi^{(n)}(\ell_P^{(n)} p)^n p^2 = 0$$

Some QG approaches predict this effective behavior

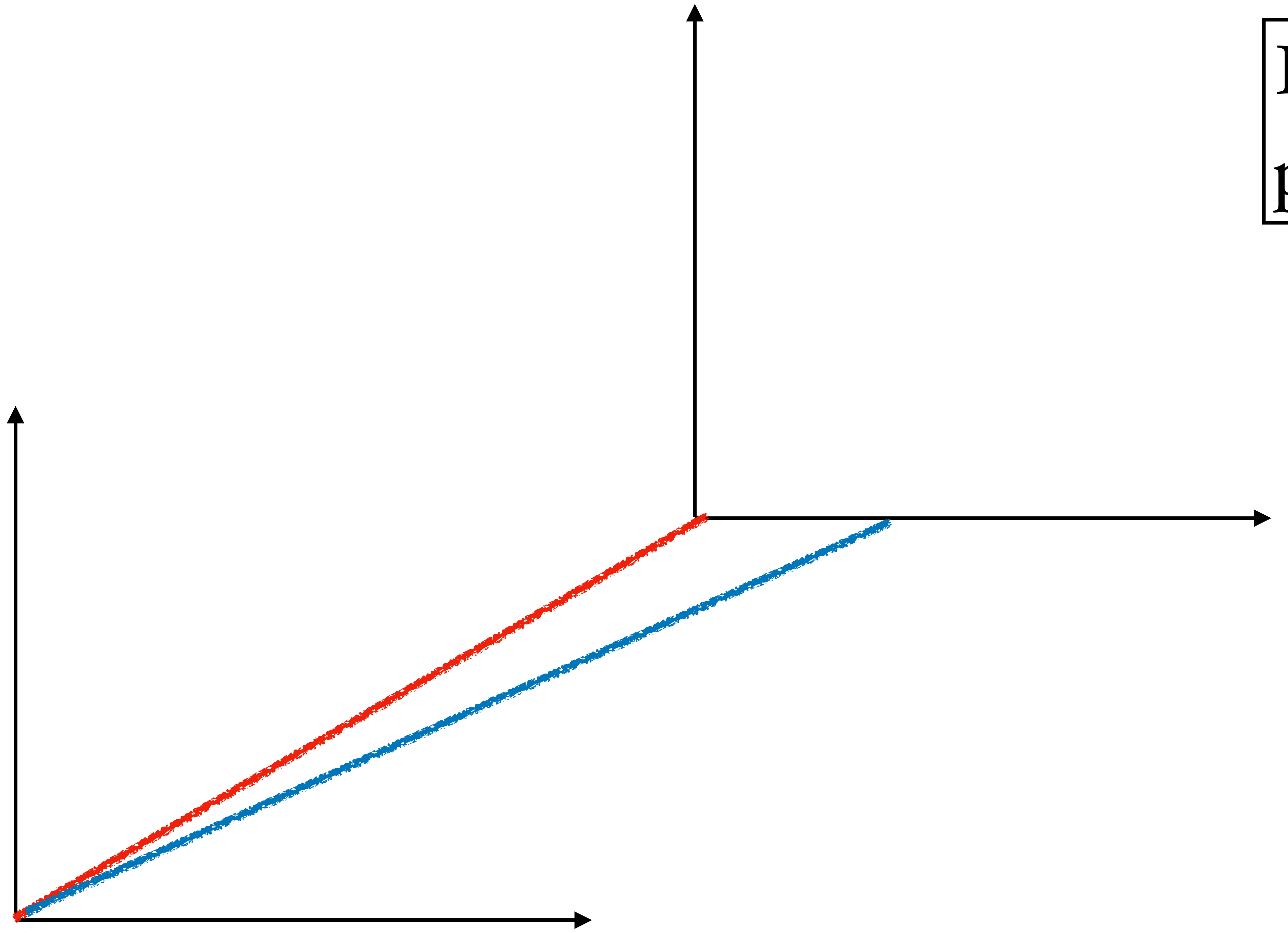
LQG: Gambini, Pullin PRD 1999, Hořava–Lifshitz: Horava PRL 2009

$$c(E) = 1 + \xi^{(n)} \frac{n+1}{2} (\ell_P^{(n)} E)^n$$

$$\Delta t = \xi \frac{1+n}{2H_0} (\ell_P^{(n)} E)^n \int_0^z \frac{(1+z')^n}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}} dz'$$

$\xi^{(1)} < 0.13$ **Vasileiou et al., PRD (2013)**

The most used formula (Jacob-Piran)



In the LIV scenario, only the trajectory of the hard photon is modified. And this produces the time delay

Assume a minimal coupling

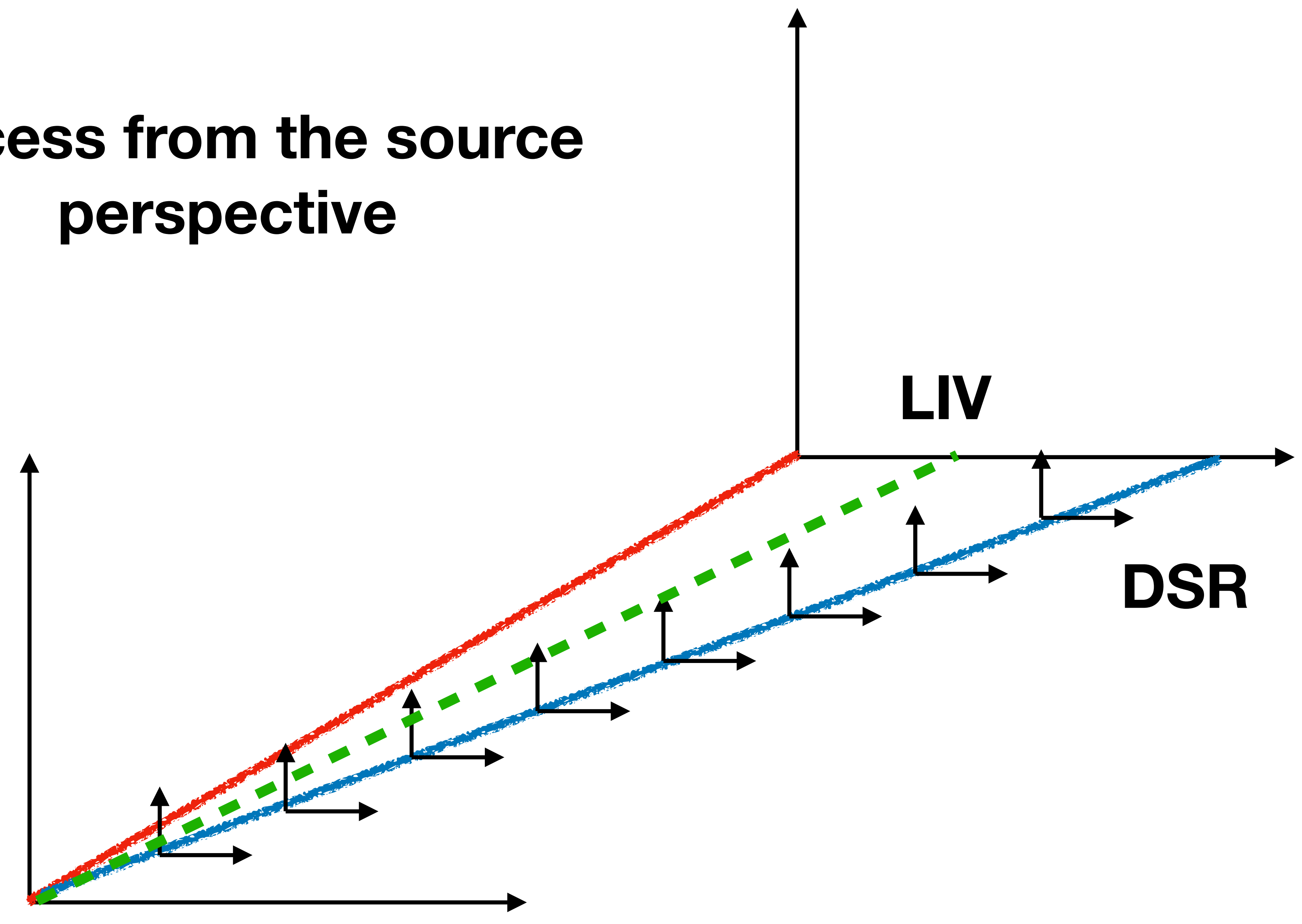
$$E^2 - \frac{p^2}{a(t)^2} - \xi^{(n)}\ell^{(n)}\frac{p^{(n+2)}}{a(t)^{n+2}} = 0$$

If one calculate the shift in the time coordinate due to the different trajectories, one finds

$$\Delta t = \xi^{(n)}\frac{1+n}{2H_0}(\ell_P^{(n)}E)^n \int_0^z \frac{(1+z')^n}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}dz'$$

However, if the translations in spacetime are locally modified, one can construct infinitesimal communications between frames until the arrival at the detector

Process from the source perspective



If in each infinitesimal step, the different frames are connected by deformed translations (since we have deformed symmetries), one finds a different formula at first order in the Planck energy

$$\Delta t = \frac{\Delta E}{M_{Pl}} \int_0^z \frac{d\bar{z}\,(1+\bar{z})}{H(\bar{z})} \left[\eta_1 + \eta_2 \left(1 - \left(1 - \frac{H(\bar{z})}{1+\bar{z}} \int_0^{\bar{z}} \frac{d\bar{z}'}{H(\bar{z}')}\right)^2\right) + \eta_3 \left(1 - \left(1 - \frac{H(\bar{z})}{1+\bar{z}} \int_0^{\bar{z}} \frac{d\bar{z}'}{H(\bar{z}')}\right)^4\right)\right].$$

Amelino-Camelia, Frattullilo, Gubitosi, Rosati, Bedic, JCAP (2024)

If $\eta_2 = \eta_3 = 0$, one finds the Jacob-Piran LIV formula

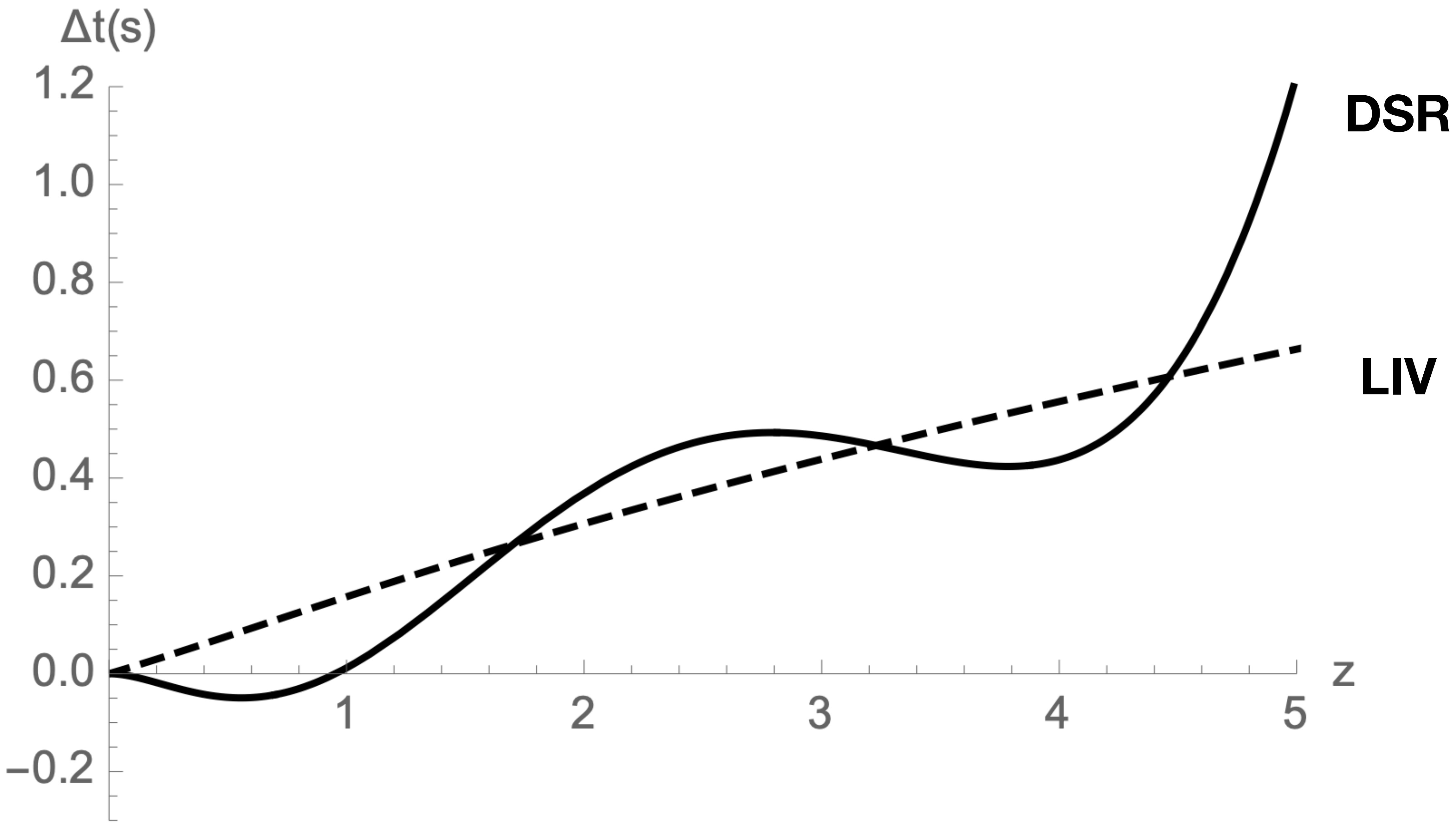
The effect of translations also accumulate over distance. That's why they are as important as the effects due to trajectories.

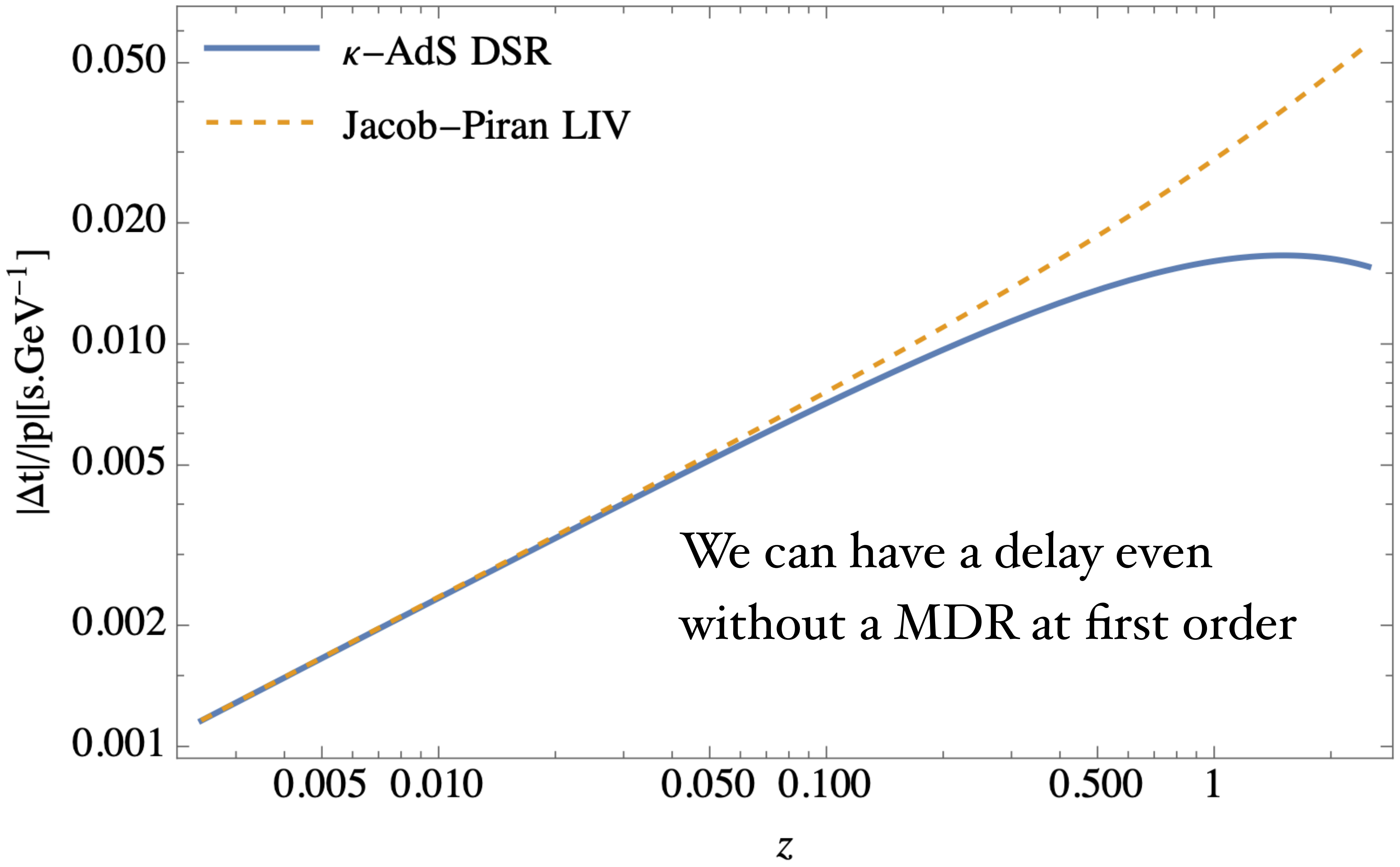
$$\Delta t = \eta \frac{\Delta E}{M_{Pl}} \int_0^z \frac{d\bar{z}(1+\bar{z})}{H(\bar{z})}$$

LIV

$$\Delta t = \frac{\Delta E}{M_{Pl}} \int_0^z \frac{d\bar{z}\,(1+\bar{z})}{H(\bar{z})} \left[\eta_1 + \eta_2 \left(1 - \left(1 - \frac{H(\bar{z})}{1+\bar{z}} \int_0^{\bar{z}} \frac{d\bar{z}'}{H(\bar{z}')} \right)^2 \right) + \eta_3 \left(1 - \left(1 - \frac{H(\bar{z})}{1+\bar{z}} \int_0^{\bar{z}} \frac{d\bar{z}'}{H(\bar{z}')} \right)^4 \right) \right].$$

DSR





This is the time delay formula for $\eta_1 = 0$, $\eta_2 = 4$, $\eta_3 = -3$

Our proposal using a different slicing based on anti-de Sitter symmetries

What is super and subluminal?

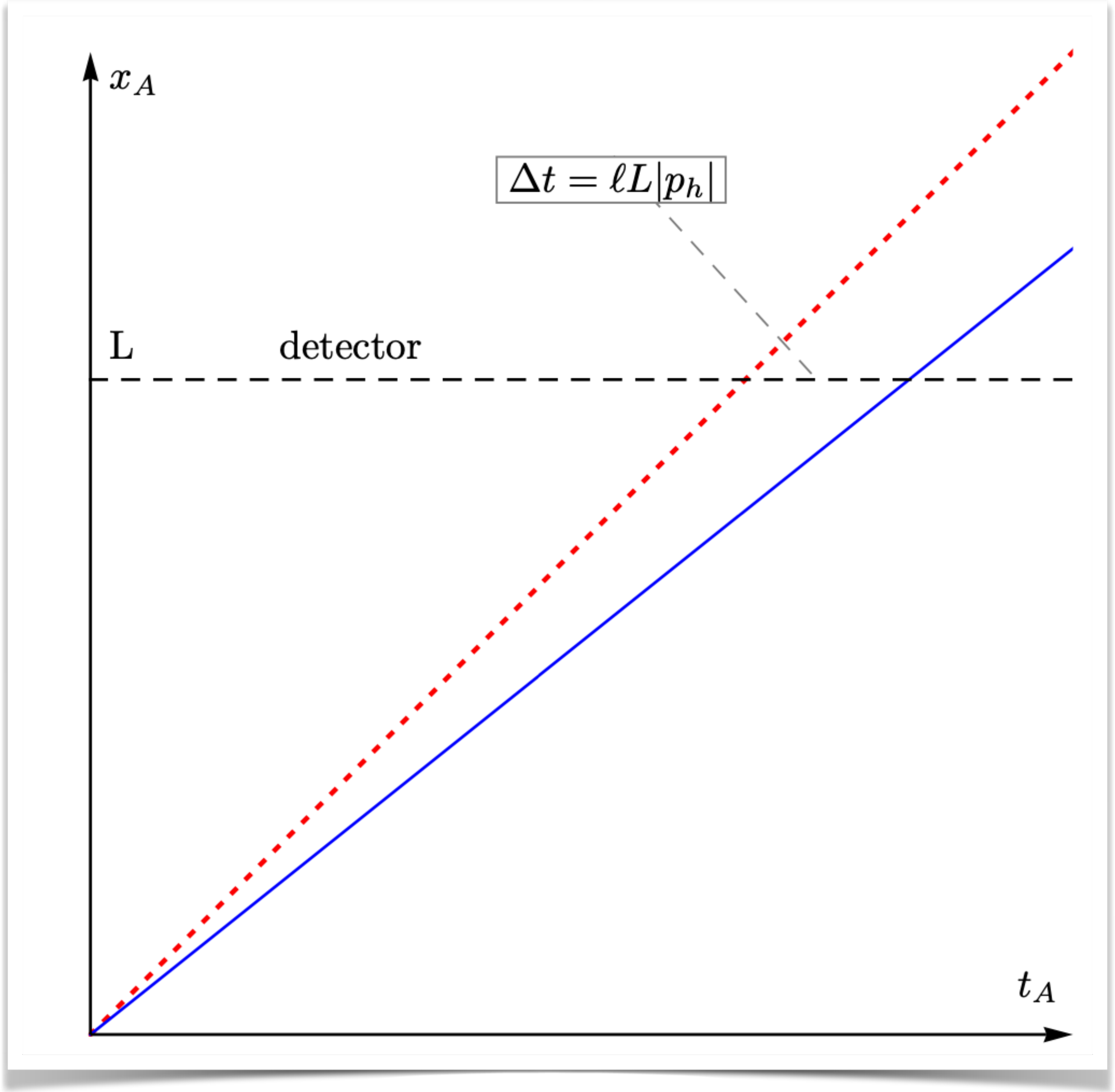
IPL, arXiv:2401.03810

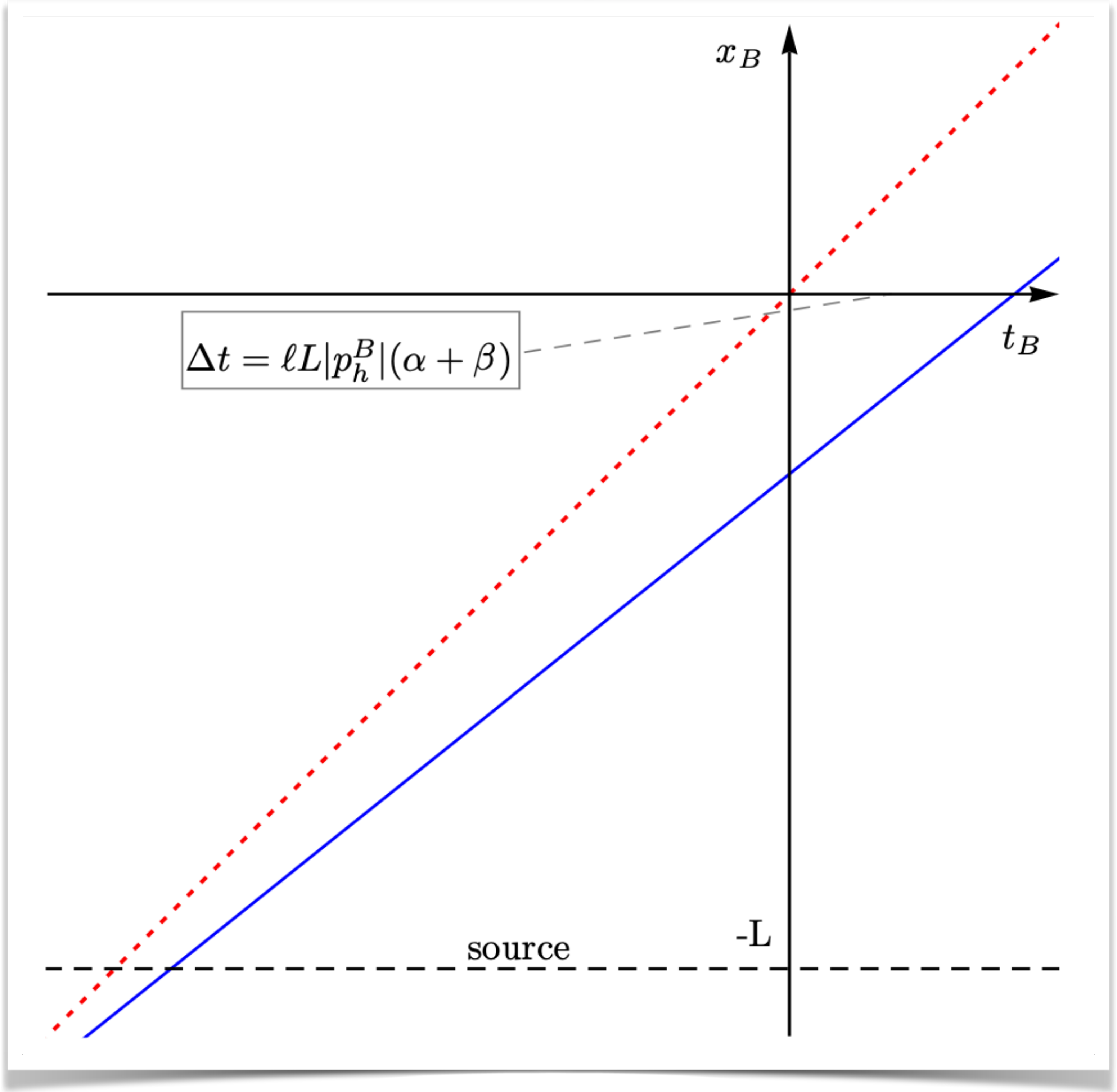
One can have a myriad of other behaviors based on different values of η_2, η_3. This gives rise to a rich phenomenology that may change the bounds currently set on the Planck scale

Mathematical curiosity

According to the coordinates of the observer that detects the time delay, the emission seems to be non-local.

The hard photon seems to be emitted at a different position (in comparison to the soft one) from the point of view of the detector frame.





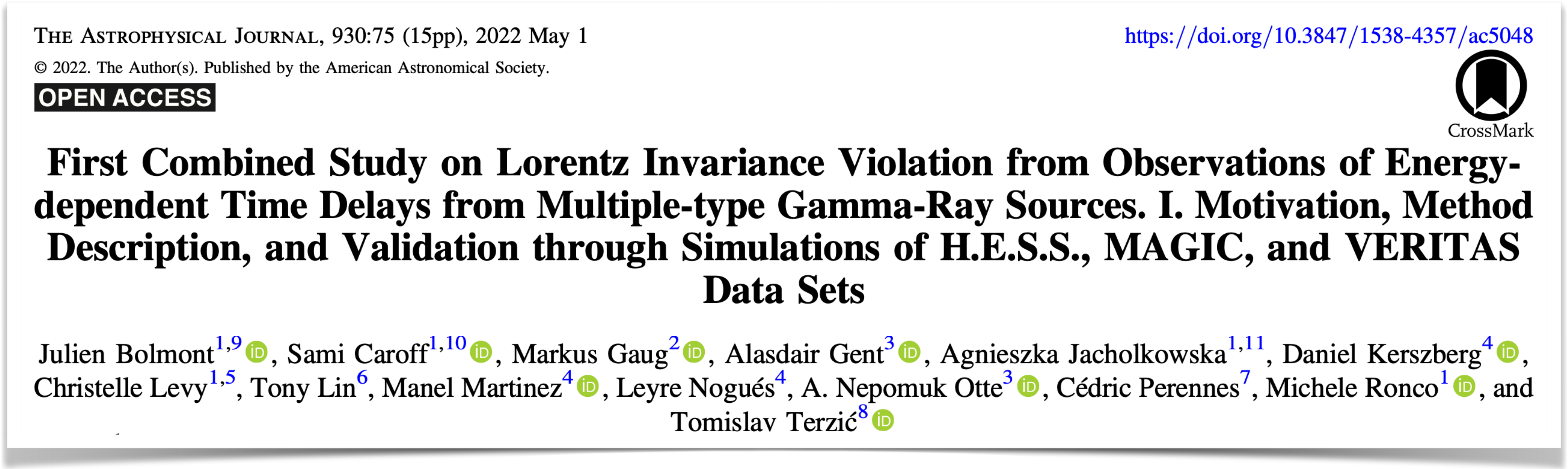
Apparent non-locality

Amelino-Camelia, Marcianò, Matassa, Rosati, PRD (2012)

We can distinguish LIV from DSR

A way to take into account these new effects are at preliminary stages of phenomenological scrutiny.

Preliminary results by other authors show that, using simulated data sets, a comparison of LIV formula and DSR formulas is realizable


THE ASTROPHYSICAL JOURNAL, 930:75 (15pp), 2022 May 1 https://doi.org/10.3847/1538-4357/ac5048
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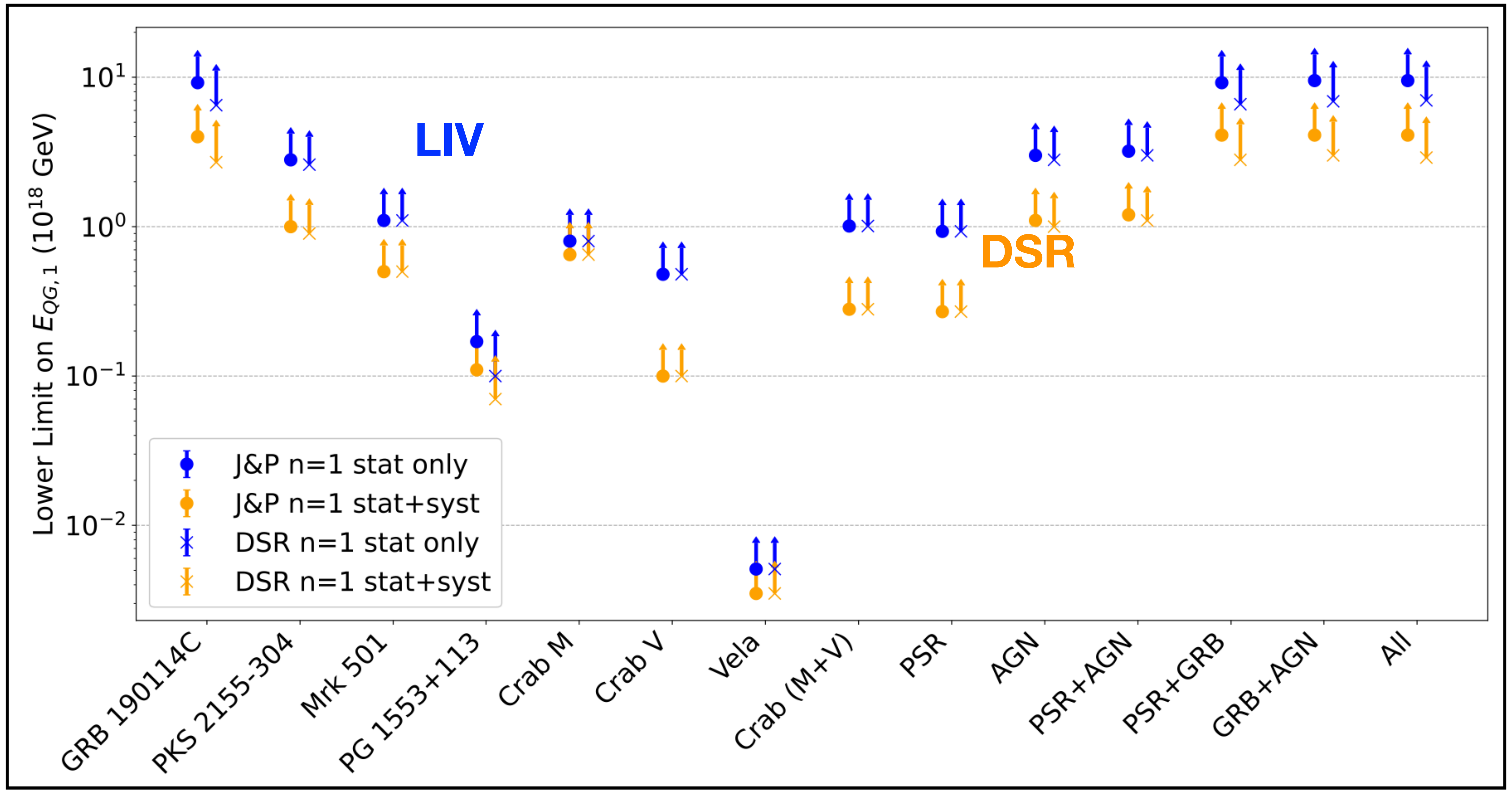
First Combined Study on Lorentz Invariance Violation from Observations of Energy-dependent Time Delays from Multiple-type Gamma-Ray Sources. I. Motivation, Method Description, and Validation through Simulations of H.E.S.S., MAGIC, and VERITAS Data Sets

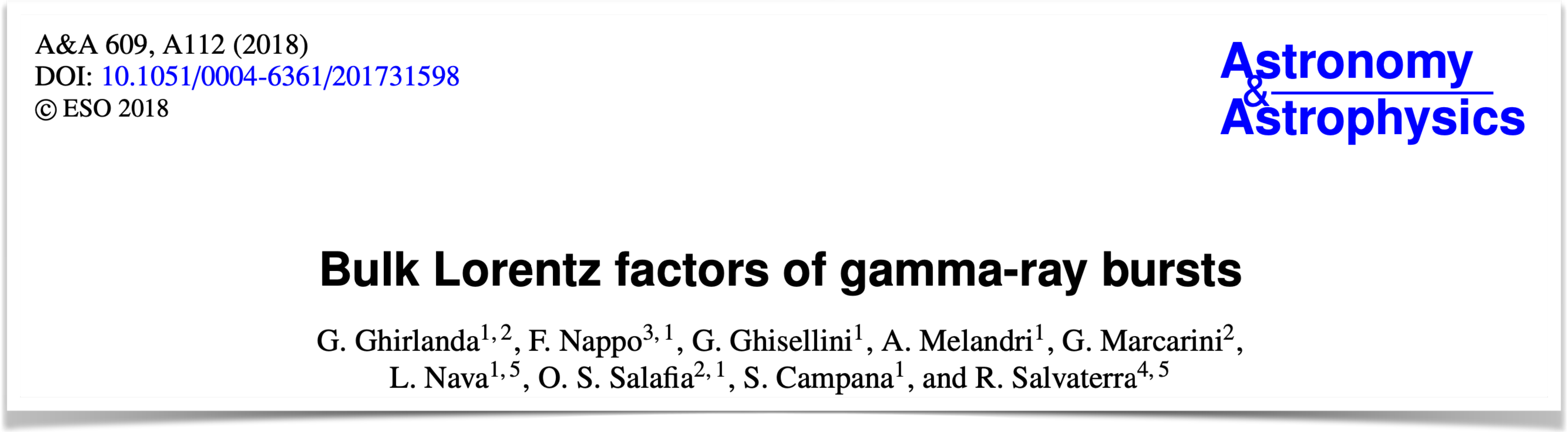
Julien Bolmont[1,9], Sami Caroff[1,10], Markus Gaug[2], Alasdair Gent[3], Agnieszka Jacholkowska[1,11], Daniel Kerszberg[4], Christelle Levy[1,5], Tony Lin[6], Manel Martinez[4], Leyre Nogués[4], A. Nepomuk Otte[3], Cédric Perennes[7], Michele Ronco[1], and Tomislav Terzić[8]



This worked analyzed only the simplest DSR case.

There's a whole new world of modifications to be analyzed

Challenge: Can deformed boosts give an extra contribution with a completely different energy/redshift dependence?


A&A 609, A112 (2018)
DOI: 10.1051/0004-6361/201731598
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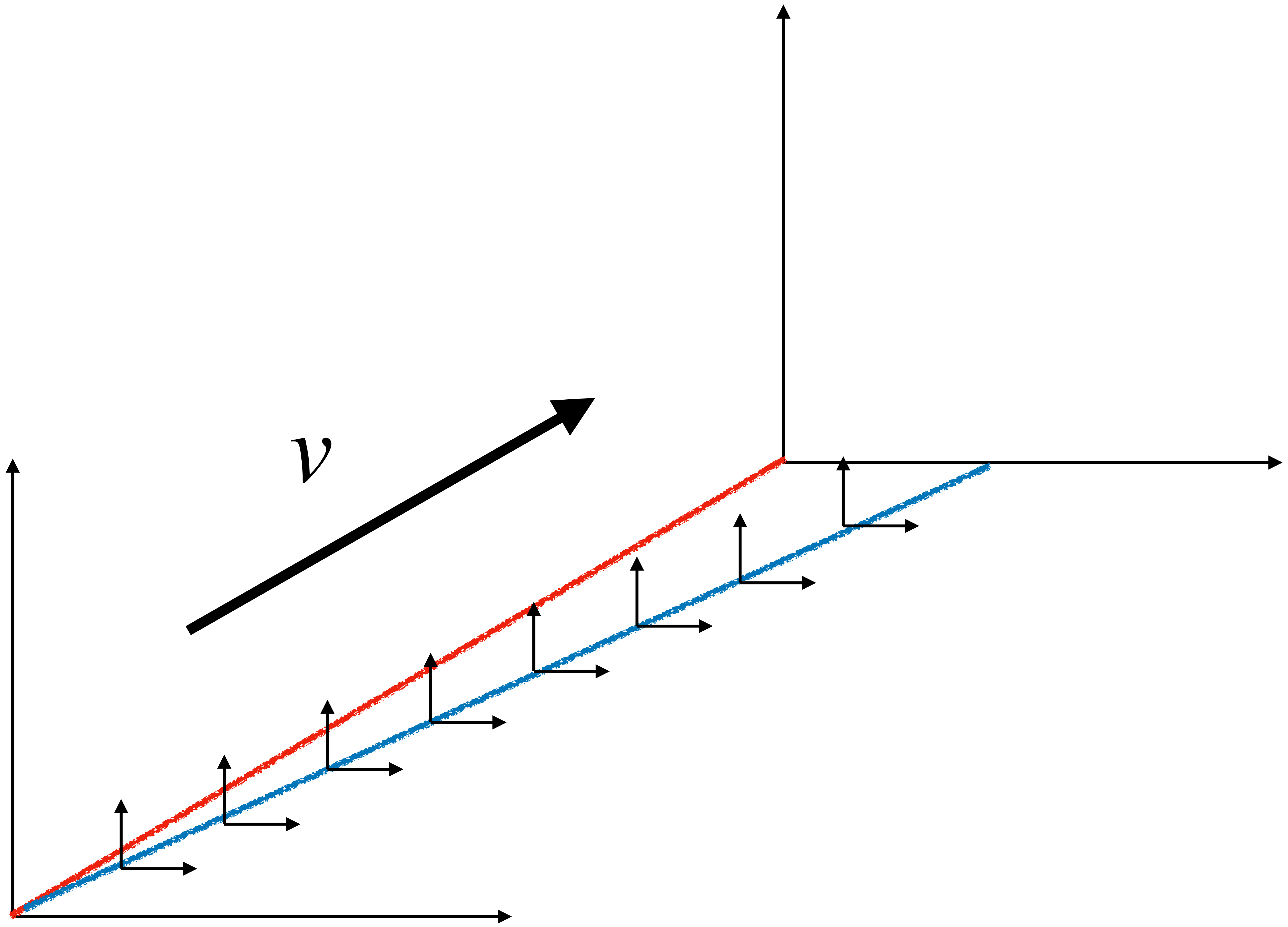
Astronomy & Astrophysics

Bulk Lorentz factors of gamma-ray bursts

G. Ghirlanda[1,2], F. Nappo[3,1], G. Ghisellini[1], A. Melandri[1], G. Marcarini[2], L. Nava[1,5], O. S. Salafia[2,1], S. Campana[1], and R. Salvaterra[4,5]



Threshold effects

Threshold effects have been the matter of discussion of Humberto's lectures, in which the expected behavior of interactions get modified in the presence of modified dispersion relations.

Also here, there are two scenarios that can be analyzed.

LIV

$$E^2 = m^2 + p^2 + s\eta^{(n)}\frac{p^{n+2}}{E_{Pl}^n}$$

$$p_\mu \oplus q_\mu = p_\mu + q_\mu$$

DSR

$$E^2 = m^2 + p^2 + s\eta^{(n)}\frac{p^{n+2}}{E_{Pl}^n}$$

$$p_\mu \oplus q_\mu = p_\mu + q_\mu + \mathcal{O}(\ell, p, q)$$

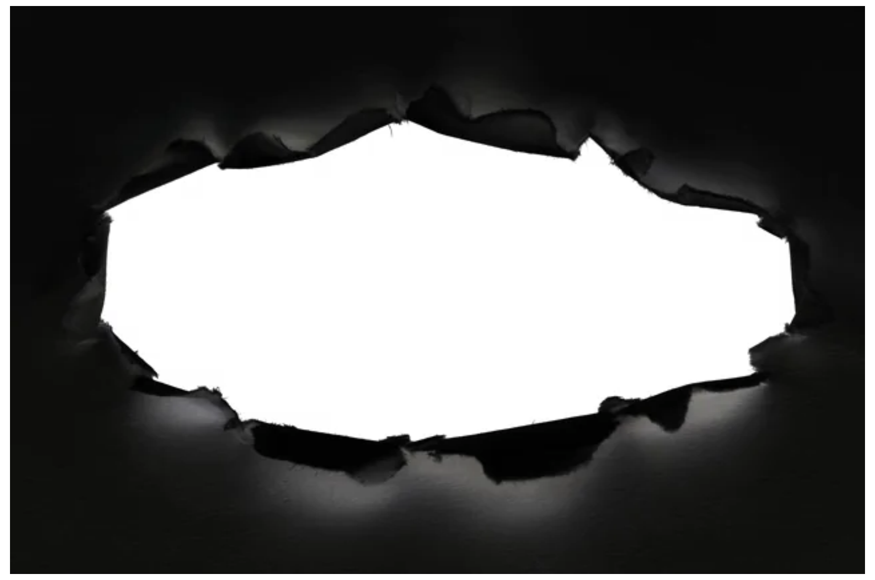
$$\Lambda_\ell(p \oplus q) = \Lambda_\ell(p) \oplus \Lambda_\ell(q)$$

LIV

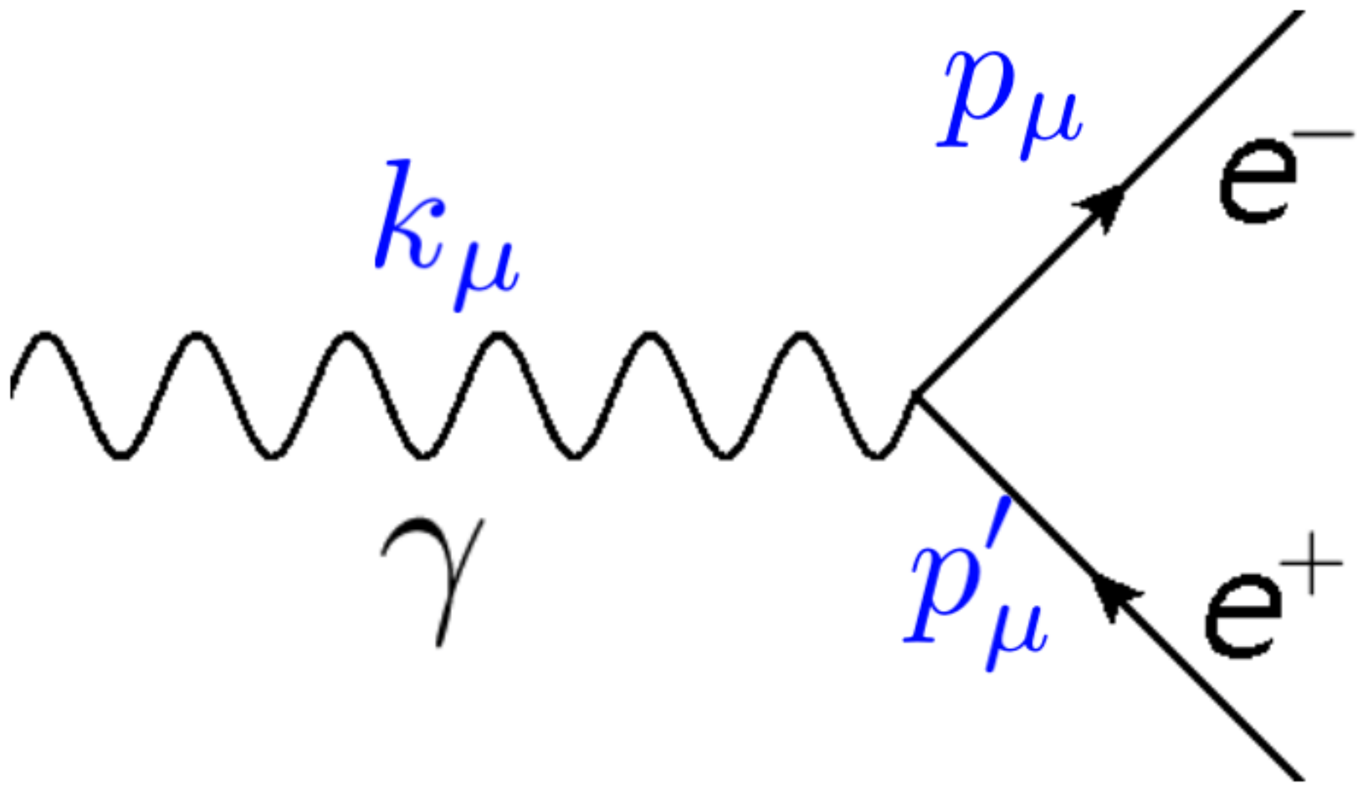
LIV has been a matter of debate in the past days, and we have discussed some processes that are forbidden in SR and become allowed in LIV.

We also can have processes that are allowed only after a certain energy threshold that is different from the SR one.

Even inducing the existence of windows with upper and lower thresholds.



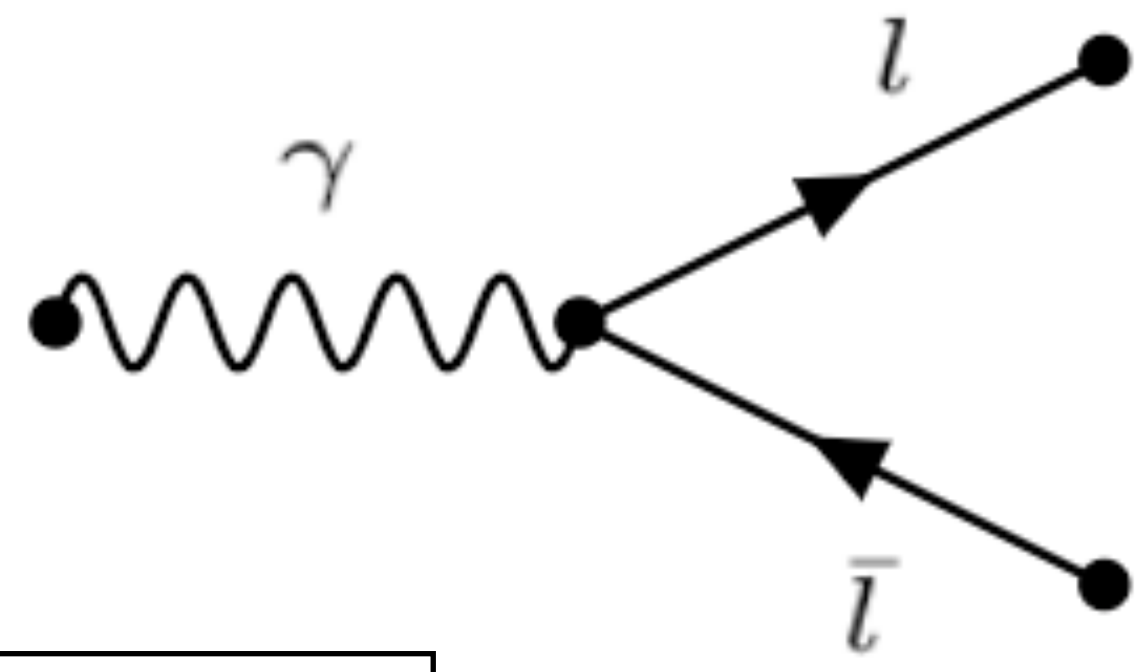
For example, photon decay becomes allowed above certain energies and the detection of photons above certain energy (meaning that they're not decaying) allows one to put strong constraints on LIV



A spacetime full of “defects” can make particles disappear

QG phenomenology in the UV - anomalous thresholds

- Threshold effects (Lorentz symmetry violation)



Constraints on Lorentz Invariance Violation from HAWC Observations of Gamma Rays above 100 TeV

A. Albert *et al.* (HAWC Collaboration)
Phys. Rev. Lett. **124**, 131101 – Published 30 March 2020

The fact that we detect photons according to SR at a such energy allows the derivation of an inequality that allows to put strong constraints

$$\xi^{(n)} \leq \frac{1}{\ell_P} \frac{(4m_e^2)^{1/n}}{E_\gamma^{1+2/n}}$$

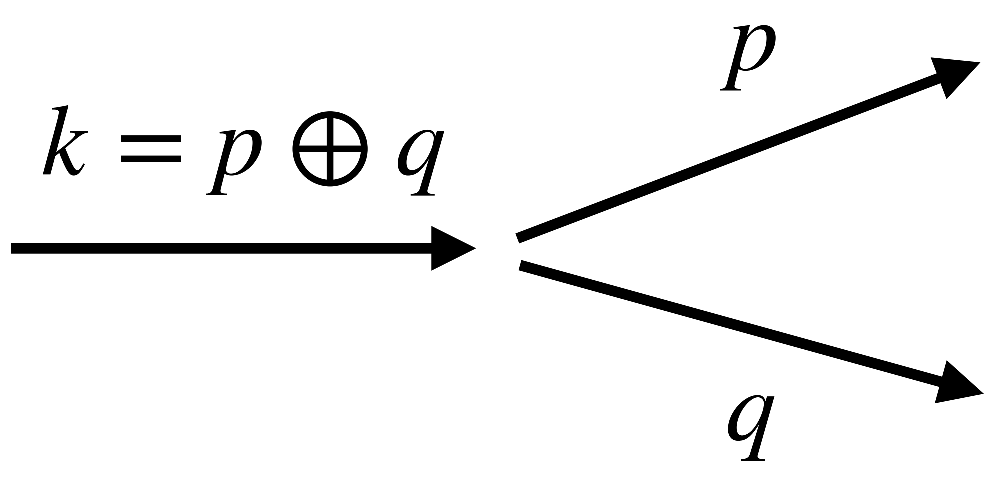
$$\xi^{(1)} \leq 5.4 \times 10^{-4}$$

$$\xi^{(2)} \leq 1.5 \times 10^{5}$$

DSR

On the other hand, if one calculates the modified threshold energies, they are negligible.

For a two-body decay, we have



$$m^2 = p_0^2 - p_1^2 - \ell p_0 p_1^2$$

$$\begin{cases} (p \oplus q)_0 = p_0 + q_0 \,, \\ (p \oplus q)_1 = p_1 + q_1 - \ell p_0 q_1 \,. \end{cases}$$

$$\pi \to \nu + \mu$$

$$E_{\mu,\nu}^{(-)} \leq E_{\mu,\nu} \leq E_{\mu,\nu}^{(+)} \,.$$

$$E_\nu^{(+)} = \frac{E_\pi(M^2 - m_\mu^2)}{M^2} + \mathcal{O}(\ell E_\pi)$$

$$E_\nu^{(-)} = \mathcal{O}(\ell E_\pi)$$

So, there's a no significant effect. This is completely different from the LIV case

This means that you don't have to modify limits of integration when calculating distributions or other quantities

IPL, Pfeifer, Morais, Alves Batista, Bezerra, JHEP (2022)

2+2 process

$$\gamma(E) + \gamma_{\mathrm{EBL/CMB}}(\varepsilon) \to e^- + e^+.$$

$\Lambda \propto E_{Pl}$

Carmona, Cortés, Pereira, Relancio, Reyes, PoS CORFU2021 (2022)

$$E_{\mathrm{th}}^{\mathrm{LIV}} = \frac{m_e^2}{\varepsilon}\left[1 + \frac{m_e^2}{\varepsilon^2}\frac{m_e^2}{\varepsilon\Lambda_{\mathrm{eff}}}\right],$$

$$E_{\mathrm{th}}^{\mathrm{DSR}} = \frac{m_e^2}{\varepsilon}\left[1 + \frac{m_e^2}{\varepsilon\Lambda_{\mathrm{eff}}}\right],$$

Amplification

As can be seen, the DSR term does not present the amplification that the LIV scenario furnishes

This same framework could be used to analyze the photon splitting $\gamma \to 3\gamma$

To analyze these effects at the Planck scale, LIV is the only appealing case phenomenologically.

Unless the parameter Λ of the DSR scenario is smaller than the Planck energy.

Time dilation

Now we know that, regarding time delays, DSR deformed translations contribute as much as the deformed trajectories (presented in LIV and DSR).

In any case, the time delay is a prediction of both LIV and DSR. Such difference would be distinguishable after the actual detection any sort of Planck scale-induced time delay.

Therefore, from a practical point of view, I would say that considering a hierarchy of experimental challenges, the mere detection of a time-delay considering the LIV scenario (Jacob-Piran formula) is the top priority

and after that, one can think about distinguishing LIV and DSR.

You may wonder what kind of effects are present due to deformed boosts.

Deformed boosts are only present in DSR. In LIV, one has the standard SR boosts.

Deformed boosts can be found from various techniques. But we have discovered a way that is very simple by the use of Finsler geometry.

Lab time

$$\tau = \int F(x, \dot{x}) d\lambda = \int \left(\sqrt{\eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} + \mathcal{O}(\ell \dot{x}) \right) d\lambda = \int \left(\sqrt{1 - v^2} + \mathcal{O}((\ell m v)^n) \right) dt$$

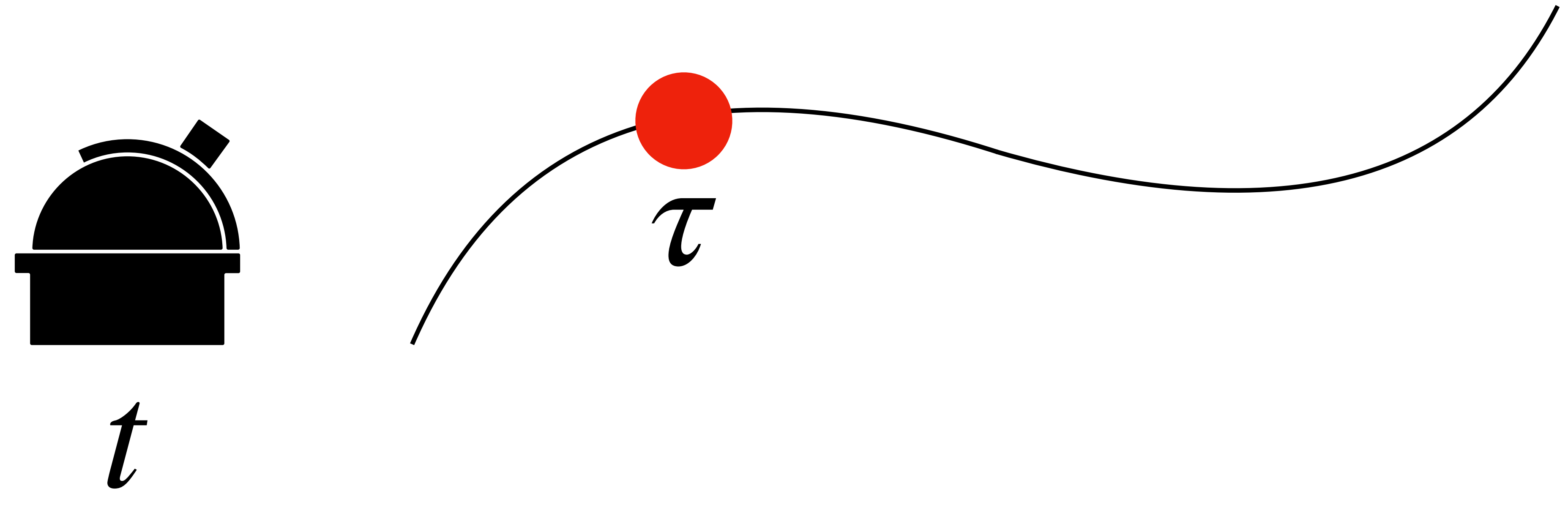
$$\tau = \gamma^{-1} \left(1 + \ell^n f(m, \gamma)\right) t \quad \Rightarrow \quad t = \gamma \tau \left(1 - \ell^n f(m, \gamma)\right)$$

t is the time measured in the lab frame
τ is the proper time
γ is the Lorentz factor that dilates this time

We checked that this is indeed the time part of a Lorentz transformation, by analyzing the geodesic equation and verifying that considering the effect on space as well, the **Finsler distance is preserved.**

Morais, IPL, Pfeifer, Alves Batista, Bezerra, PLB (2024)

You can consider that the **proper time** is the **decay time of a particle at rest**



If you use the deformed Lorentz transformation between energy and momentum, that can also be found using a very simple Finsler technique

$$E = m\frac{\partial F}{\partial \dot{x}^0} = m\gamma + \ell^n g(m,\gamma)$$

$$\gamma = \frac{E}{m}\left(1 + \ell^n h(E,m)\right)$$

Plugging these expressions into the previous formula, we find an expression that shows a deformation of the time dilation due to Planck scale corrections

$$t = \frac{E}{m}\tau\left(1 + \ell^n G(E,m)\right)$$

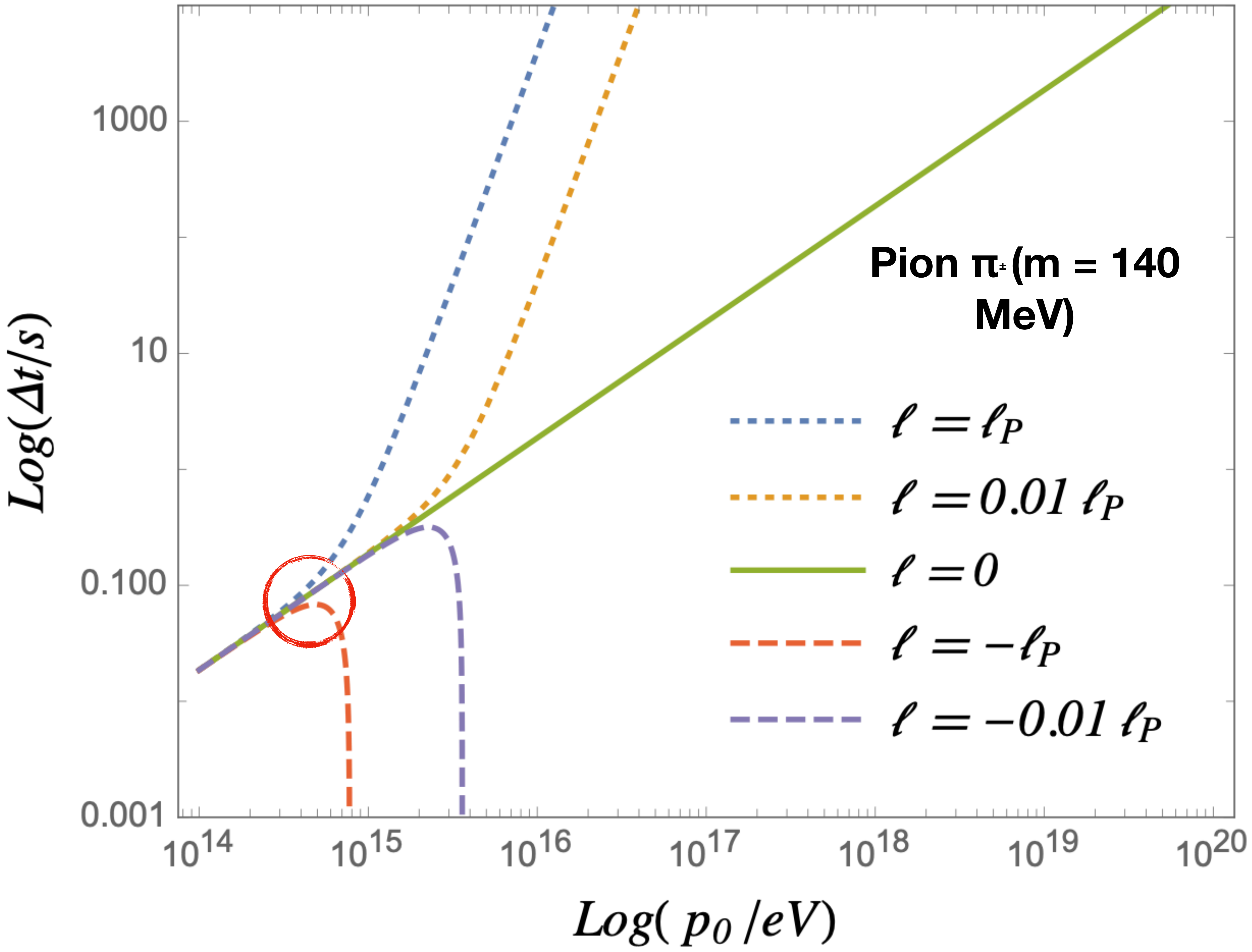
Why is this relevant?

Consider $\ell^n = \eta^{(n)} E_{Pl}^{-n}$

Amplifier

$$E^2 - p^2 = m^2 + \eta^{(n)} \frac{|p|^{n+2}}{E_{\rm Pl}^n}$$

$$t = \gamma_{\rm CP}\tau = \frac{E}{m}\left[1 + \frac{n\eta^{(n)}}{2}\left(\frac{|p|}{m}\right)^2\left(\frac{|p|}{E_{\rm Pl}}\right)^n\right]\tau\,.$$



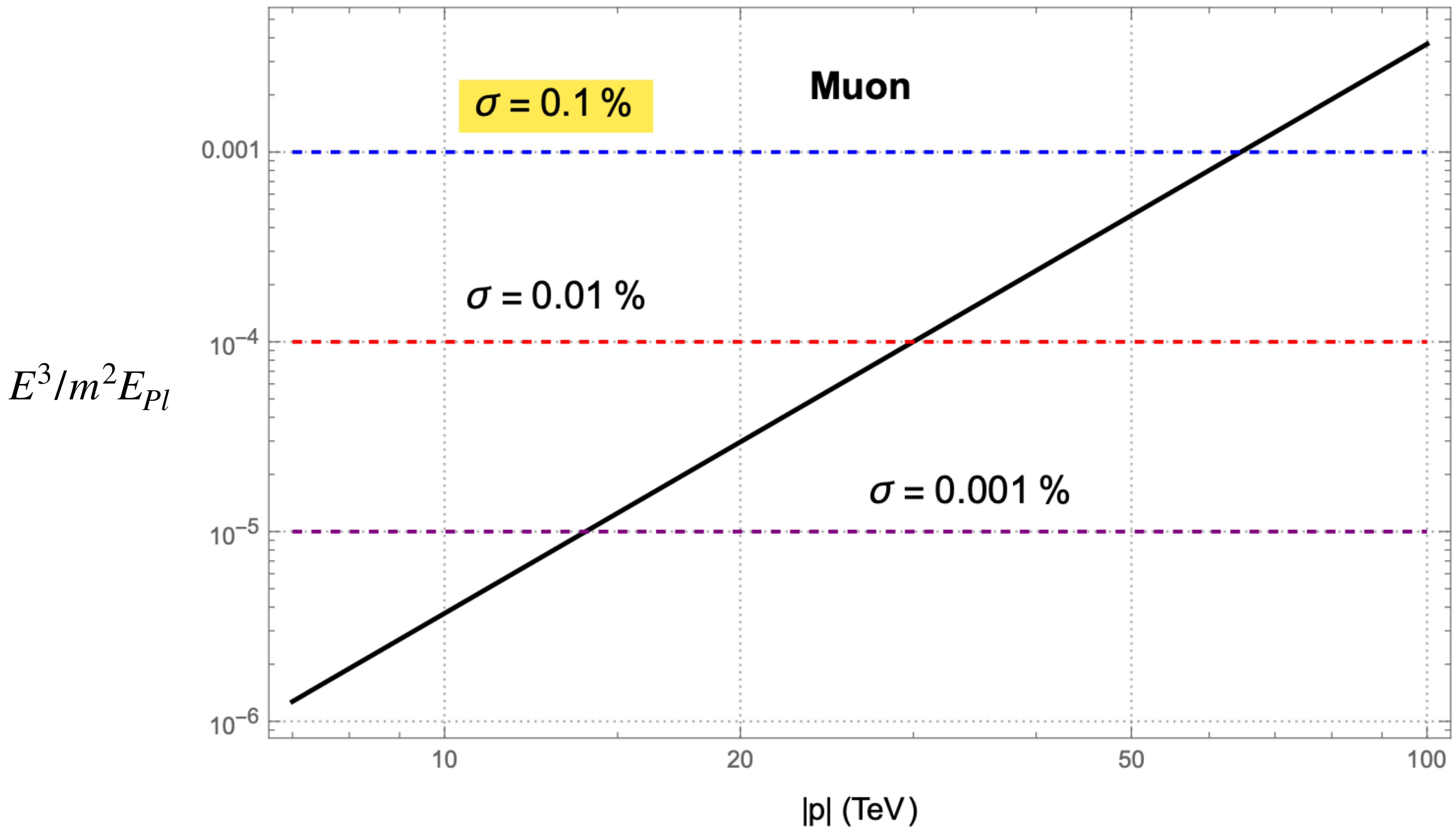
For $E = 100\,\text{TeV}$, $m = 100\,\text{MeV}$, $E_{Pl} = 10^{28}\,\text{eV}$

We find that $\left(\frac{E}{m}\right)^2 \frac{E}{E_{Pl}} = 1\,\%$

The most natural candidate to test this effect is UHECR

In fact, to test this effect in an accelerator, we would need a $10+$ TeV muon collider and improvement in precision of measurements in 2 orders of magnitude

IPL, Pfeifer, CQG (2023)



Based on an entirely different intuition, a group based in L'Aquila arrived at similar formulas and has done some **preliminary** investigations on this possibility, presented on proceedings

P. Abreu et. al.[Pierre Auger Collaboration], PoS (ICRC) 2021, doi:10.22323/1.395.0340

Trimarelli [Pierre Auger], EPJ Web Conf., (2023) doi:10.1051/epjconf/202328305003

A problematic issue of this analysis consists in the substitution of the Lorentz factor by an effective one that, at dominant order, is very similar to the strong one that was presented previously

$$\gamma_{eff} \approx \frac{E}{m}\left(1 + \eta \frac{E^3}{m^2 E_{Pl}}\right)$$

The problem with this substitution everywhere is that there are some expressions that in SR can be written with γ, but that shouldn't suffer such a strong modification in this scenario

For example

$\beta = \dfrac{p}{m\gamma}$ is the speed of the particle that should be $\beta \approx \dfrac{p}{E}\left(1 + \eta \dfrac{p}{E_{Pl}}\right)$. So, without amplification.

Although it's a preliminary investigation, it shows great results for this observable

Looks for the fluctuation in the number of muons to find $-10^{-1} < \eta^{(1)} < 5.95 \times 10^{-6}$

Planck scale sensitivity

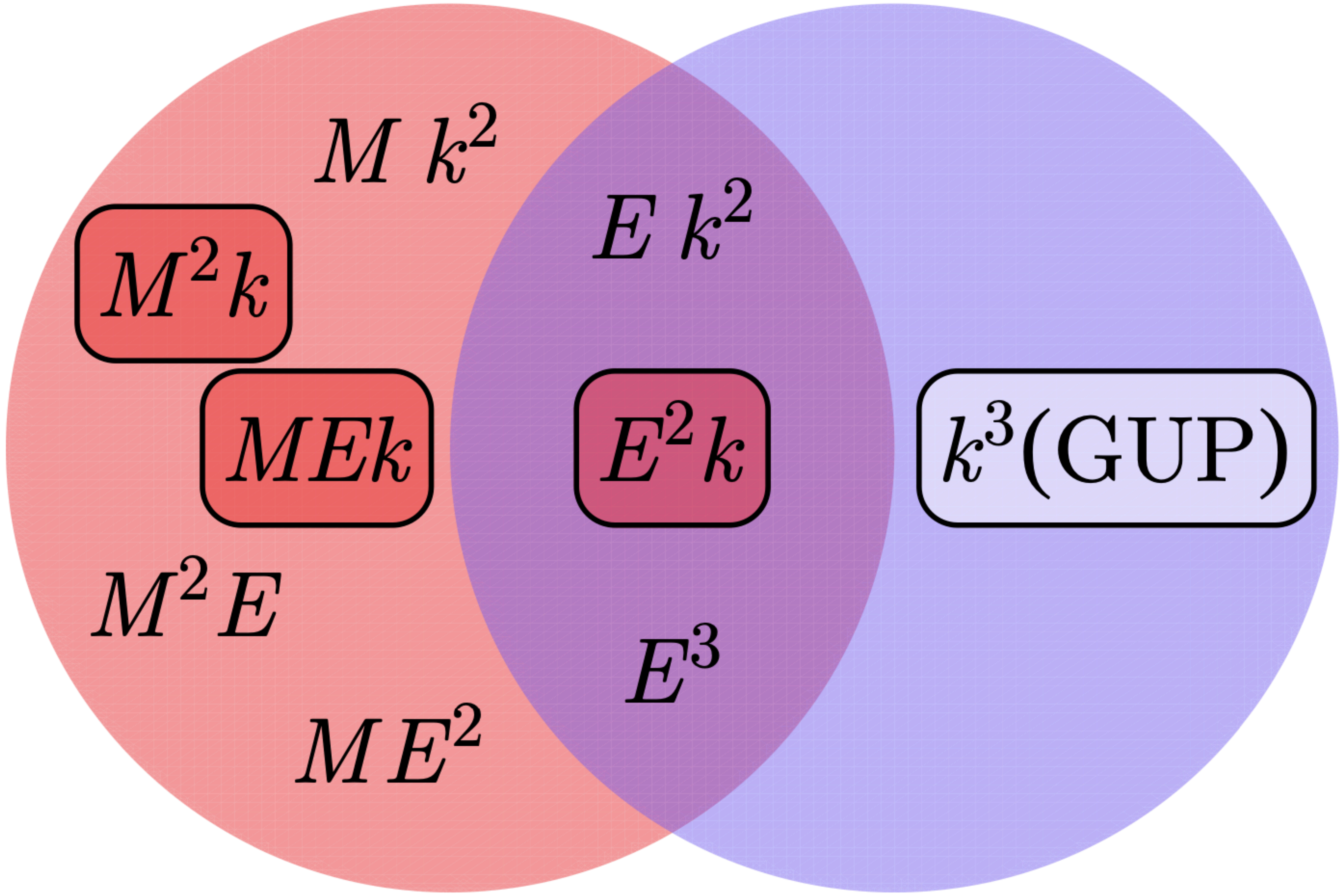
Lesson I : All the effects that present an intrinsic boost (in time and energy) should be modified.

Lesson II : In the DSR scenario, threshold energies modifications are much smaller that the boost correction. Therefore, it's not necessary to change limits of integration in energy. You can proceed to just change the boost.

Lesson III : Deformed trajectories are also not relevant because the massive particles that decay don't travel long enough.

Alternative MDRs

$$M^2c^4 = E^2 - k^2c^2 + \ell \sum_{n=0}^{2} \sum_{m=0}^{3-n} a_{n,m,3-n-m} (Mc)^n k^m \left(\frac{E}{c}\right)^{3-n-m},$$



Wagner, Varão, Lobo, Bezerra, PRD (2023)

FIG. 1: Dependence of correction terms to the relativistic dispersion relation on momentum k, mass M, and energy E for all models at order ℓ. The violet (left) and red (right) circles indicate those which are amenable to relativistic (UV) and nonrelativistic (IR) measurements, respectively. Those which are particularly suitable for nonrelativistic reasoning are framed and have an additional red background. The GUP is indicated by a frame and with a white background.

QG phenomenology in the IR - cold atoms

- **Recoil frequency of cold atoms involving Raman transitions**

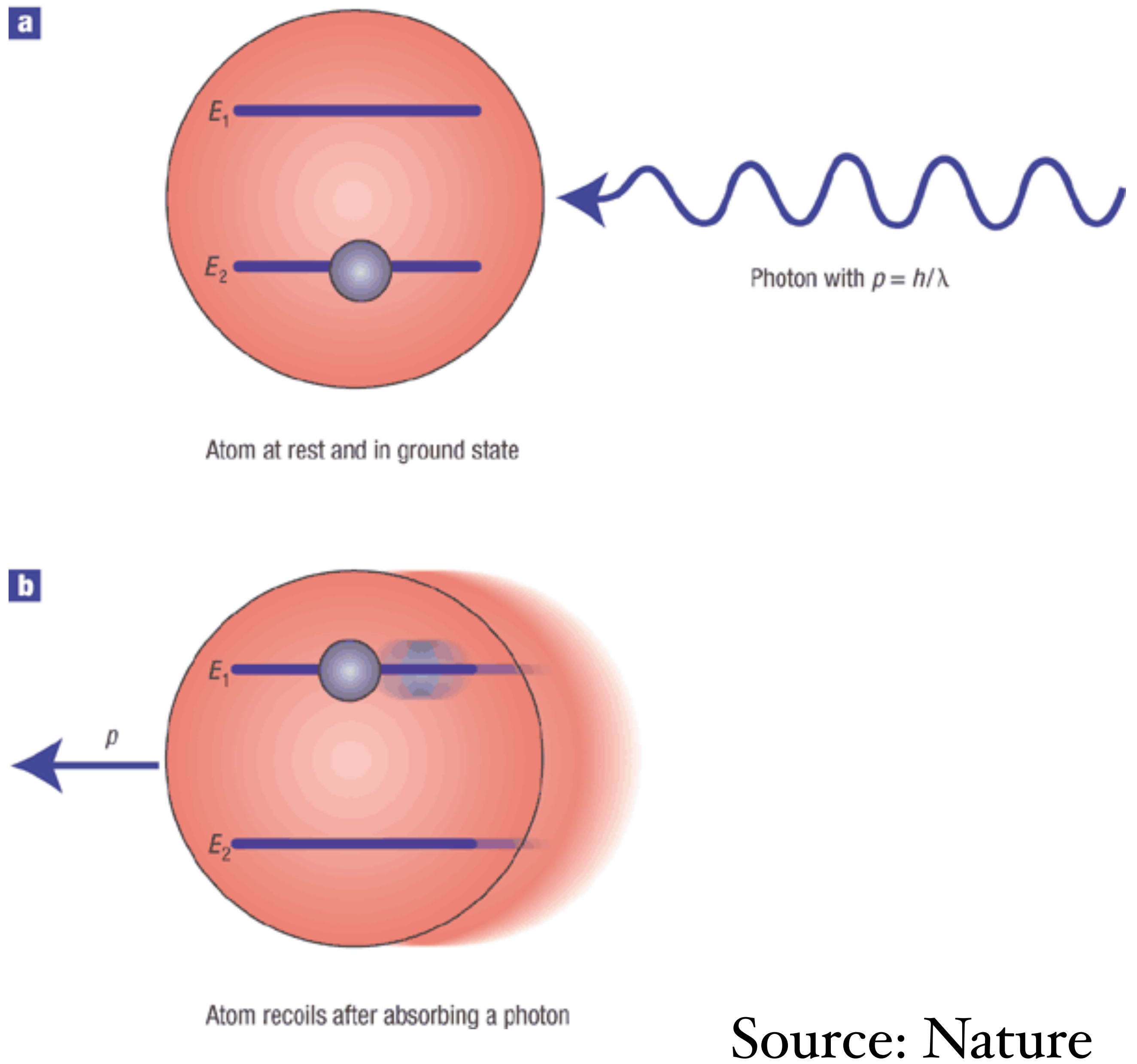
$$E \simeq m + \frac{p^2}{2m} + \frac{1}{2M_P}\left(\xi_1 mp + \xi_2 p^2 + \xi_3 \frac{p^3}{m}\right)$$

$$\frac{\Delta\nu}{2\nu_*(\nu_*+p/h)}\left[1-\xi_1\left(\frac{m}{2M_P}\right)\left(\frac{m}{h\nu_*+p}\right)\right]=\frac{\alpha^2}{2R_\infty}\frac{m_e}{m_u}\frac{m_u}{m}$$

Atom momentum

10^9

Atomic mass



Source: Nature

$$-6.0 < \xi_1 < 2.4$$

Constraining the Energy-Momentum Dispersion Relation with Planck-Scale Sensitivity Using Cold Atoms

Giovanni Amelino-Camelia, Claus Lämmerzahl, Flavio Mercati, and Guglielmo M. Tino

Phys. Rev. Lett. **103**, 171302 – Published 21 October 2009; Erratum Phys. Rev. Lett. **104**, 039901 (2010)

In the non relativistic limit, we derived a modified Schrödinger equation

Metric

Laplacian minimally coupled with vector potential for electromagnetism and gravity

Scalar potentials of electromagnetism and gravity

$$\hat{H}_{\ell,\mathrm{NR,EM}}\psi = \Bigg[-\frac{h^{ij}}{2M}\left(\nabla_i - ieA_i^e - iMA_i^g\right)\left(\nabla_j - ieA_j^e - iMA_j^g\right) + e\phi_e + M\phi_\mathrm{g} - \frac{\ell}{2M}\sum_{n=1}^{3}\xi_n (Mc)^{3-n}\left[-h^{ij}\left(\nabla_i - ieA_i^e - iM\mathcal{A}_i\right)\left(\nabla_j - ieA_j^e - iM\mathcal{A}_j\right)\right]^{\frac{n}{2}}\Bigg]\psi.$$

Wagner, Varão, Lobo, Bezerra, PRD (2023)

If we use the Schwarzschild metric and approximate the gravitational field in the vicinity of the surface of the spherical massive object, we can derive a Hamiltonian of the kind

$$\hat{H}_{\ell,\mathrm{NR,EM}} = \frac{\hat{k}^2}{2M} + Mgz - \frac{\ell}{2M}\sum_{n=1}^{3}\xi_n (Mc)^{3-n}\hat{k}^n_{\mathcal{A}},$$

$\mathscr{A}$ is the velocity of the earth relative to the rest frame of the CMB $\mathscr{A} \sim 10^{-3}c$

$$\hat{H}_\ell = \frac{\hat{k}^2}{2M_I(\xi_n)} + Mgz$$

All the modifications are inserted into an effective inertial mass M_I

This indicates a deformation of the expected acceleration $a = <\ddot{x}>$

$$M_{\mathrm{I}}a = -Mg \equiv M_{\mathrm{g}}g,$$

Violation of the weak equivalence principle may be tightly constrained due to the high experimental precision Eötvös-like experiments can achieve nowadays

QG phenomenology in the IR - equivalence principle

$$\hat{H}_{\ell,\mathrm{NR,EM}} = \frac{\hat{k}^2}{2M} + Mgz - \frac{\ell}{2M}\sum_{n=1}^{3}\xi_n (Mc)^{3-n}\hat{k}^n_A. \quad = \frac{\hat{k}^2}{2M_I} + M_g g z$$

Fabian Wagner, Gislaine Varão, **IPL**, Valdir. B. Bezerra, "Quantum-spacetime effects on nonrelativistic Schrödinger evolution", PRD 108 (2023)

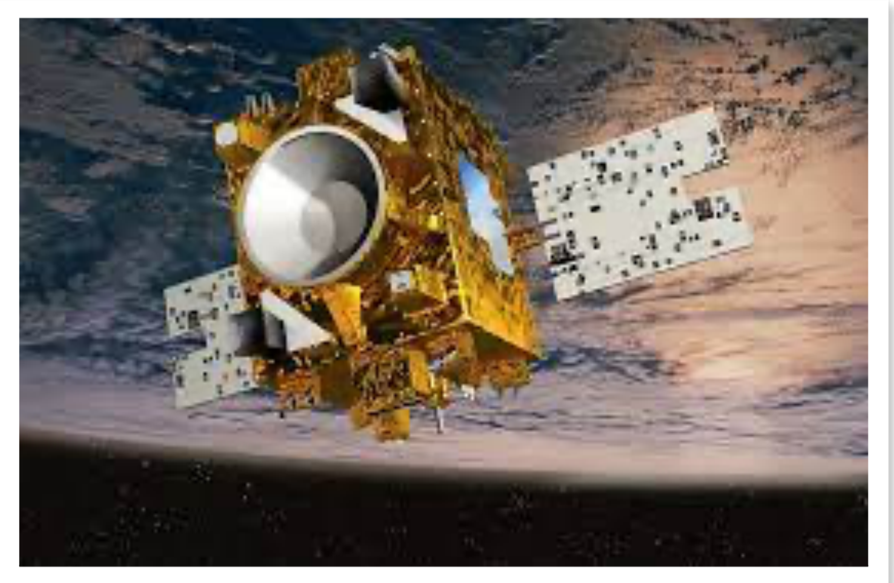
$$M_I \approx M_g\left[1 + \frac{\ell M_g c}{2}\left(10^3\xi_1 + \xi_2 + 10^{-3}\xi_3\right)\right]$$

Inertial mass ≠ Gravitational mass

Considering free fall of two distinct massive bodies A and B, this **violation of the weak equivalence** principle can be summarized in the **Eötvös parameter**

$$\eta(A,B) = 2\left\langle \frac{\frac{M_{g,A}}{M_{I,A}} - \frac{M_{g,B}}{M_{I,B}}}{\frac{M_{g,A}}{M_{I,A}} + \frac{M_{g,B}}{M_{I,B}}} \right\rangle$$

The MICROSCOPE Collaboration gives $|\eta| < 10^{-14}$



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MICROSCOPE Mission: Final Results of the Test of the Equivalence Principle

Pierre Touboul *et al.* (MICROSCOPE Collaboration)
Phys. Rev. Lett. **129**, 121102 – Published 14 September 2022

Physics See Viewpoint: Satellite Confirms the Principle of Falling

$|\xi_1| \leq \mathcal{O}(10^1)$

$|\xi_2| \leq \mathcal{O}(10^4)$

$|\xi_3| \leq \mathcal{O}(10^7)$

Conclusion

- It would be impossible to address all the phenomenologically interesting effects, but I made a small selection of very popular areas and included some others that I think deserve further investigation.

- Again, I strongly recommend the reviews cited in the first lecture.

- I hope this little journey through the description of some observables and the discussion of some subtleties of deviations from Lorentz symmetry has been interesting for you.

Robert Gilmore, “Alice in Quantumland”, Copernicus Books, 1995

Thank you!
Obrigado!