UNIVERSIDADE DE SÃO PAULO
Instituto de Física de São Carlos

INSPIRANDO TU MEJOR VERSIÓN

UDEM

LIV Class 3

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Module ~~2~~ 3: Theoretical Frameworks for Lorentz Symmetry Violation

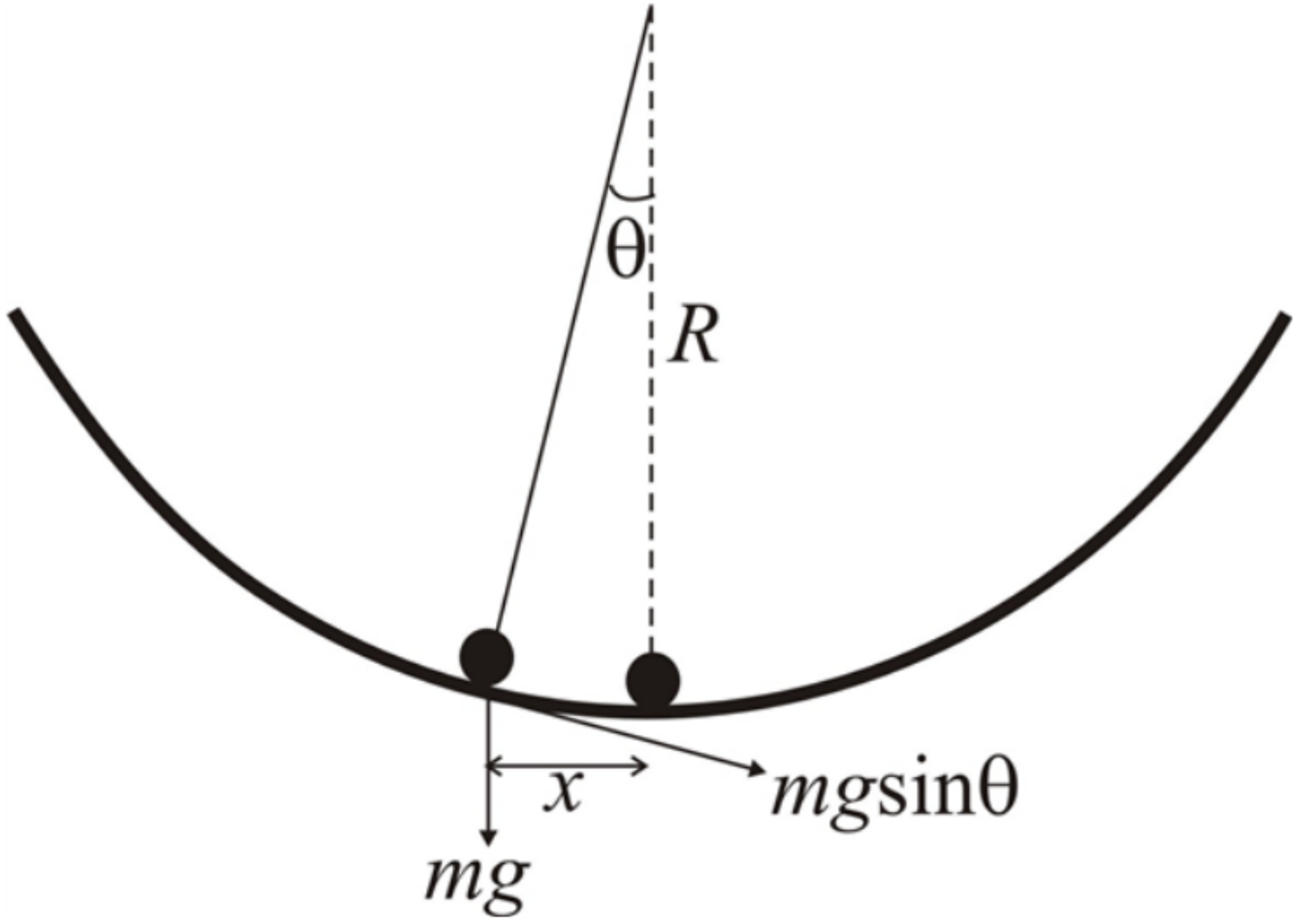
3.1 Standard Model Extension (SME)

- Introduction to the Standard Model Extension as a framework for incorporating Lorentz symmetry violation
- Parameters, implications and comparison with MDR

2.2 Alternative Theories -> DSR (Dr. Iarley Pereira Lobo)

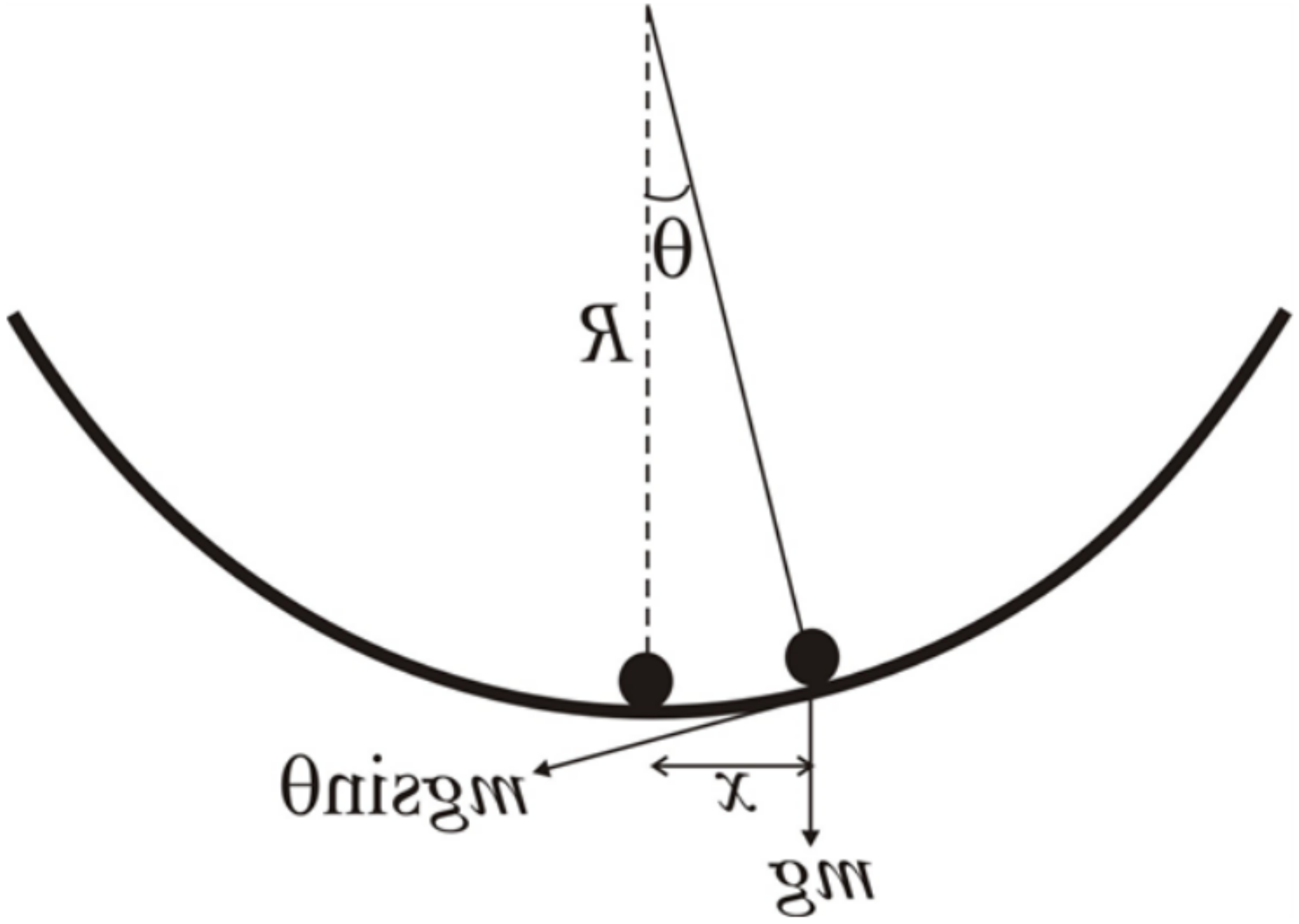
- Overview of alternative theories proposing Lorentz symmetry violation
- String theory, quantum gravity, and other beyond-the-Standard-Model approaches

Trajectories



…and if we invert it

Trajectories



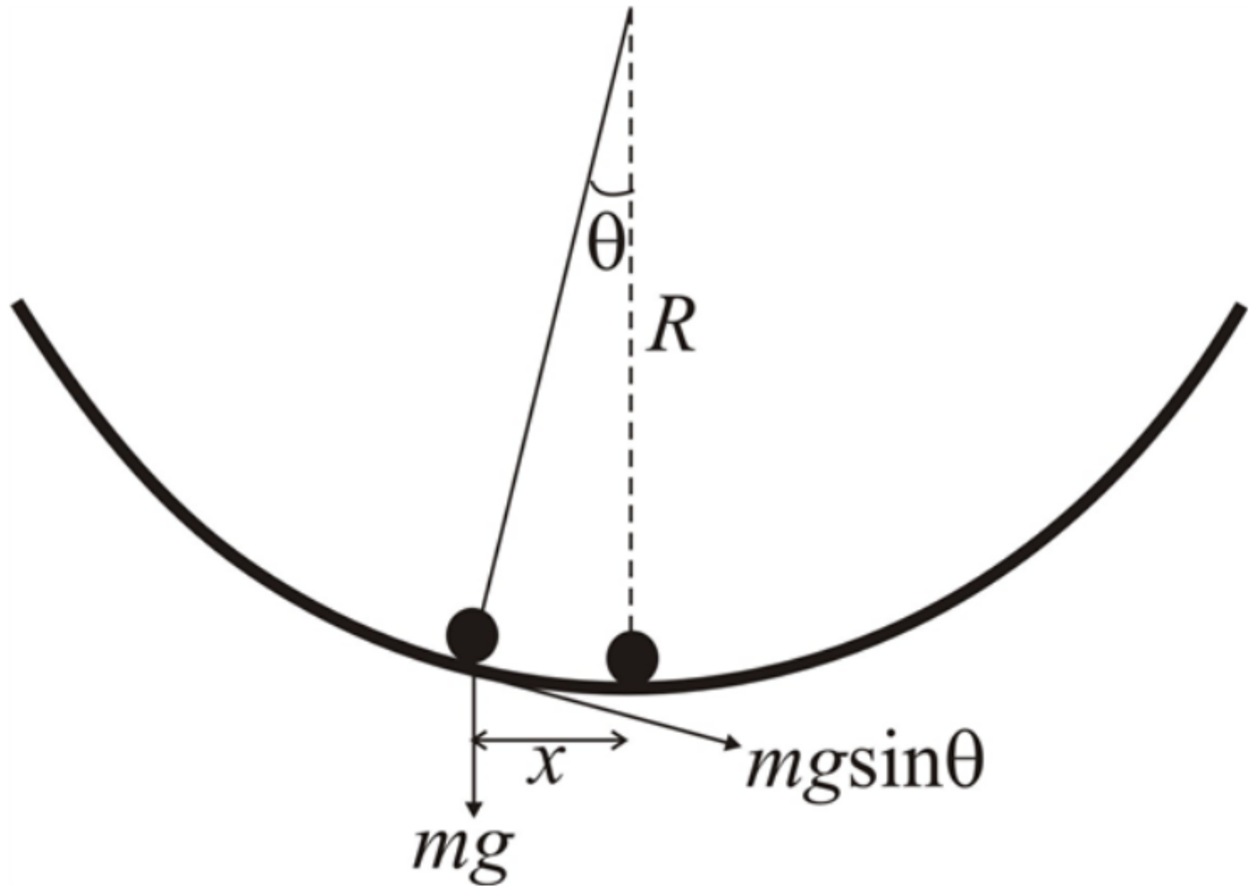
...and if we invert it

Trajectories

- There is a "natural" trajectory
- Trajectories can be described mathematically (functions), e.g.

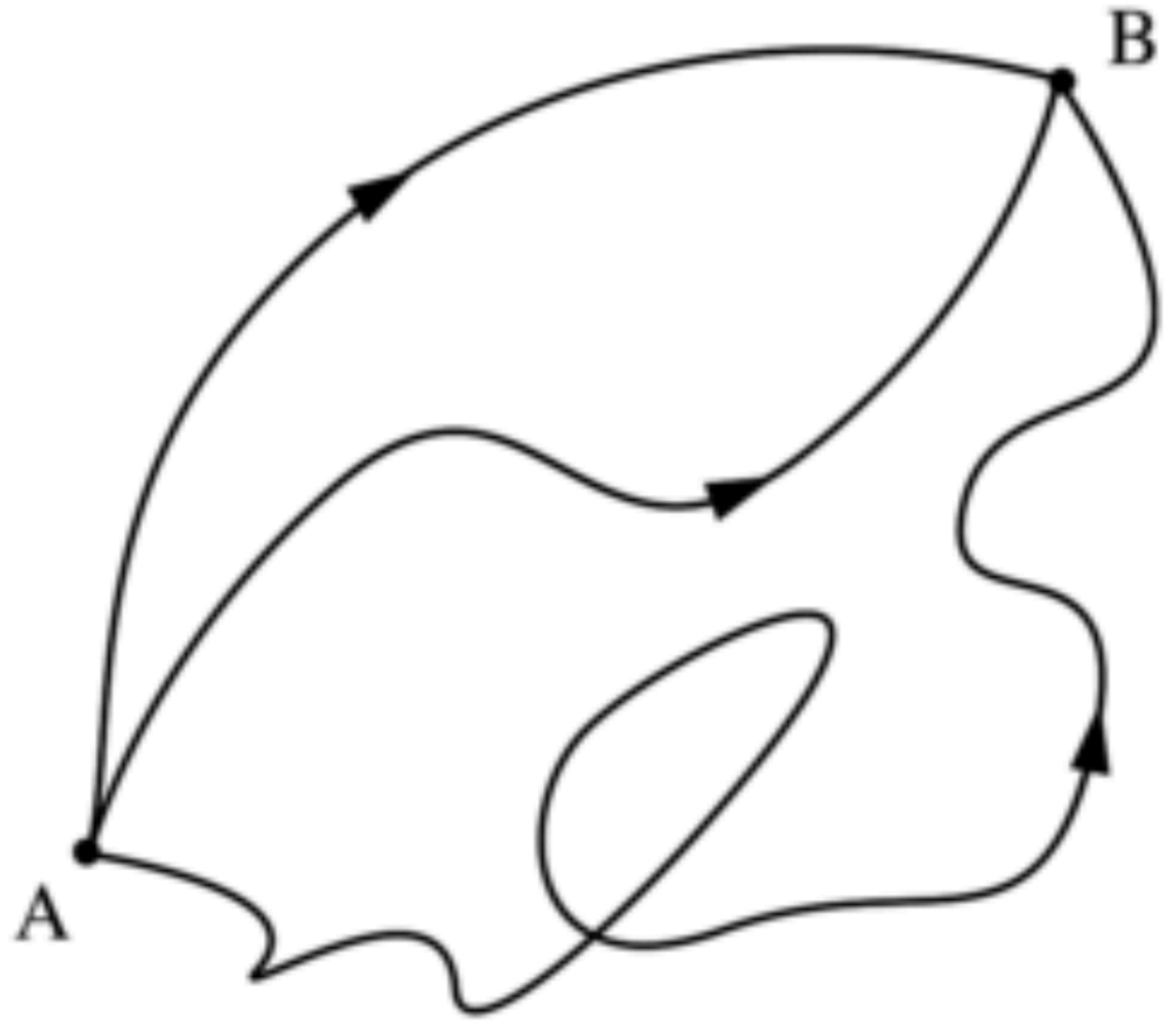
$$\frac{d^2 y(t)}{dt^2} = a \rightarrow y(t) = y_o + v_{o_y}\ t + 1/2\ a\ t^2$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$



- …and they can also have symmetries

Lagrangian

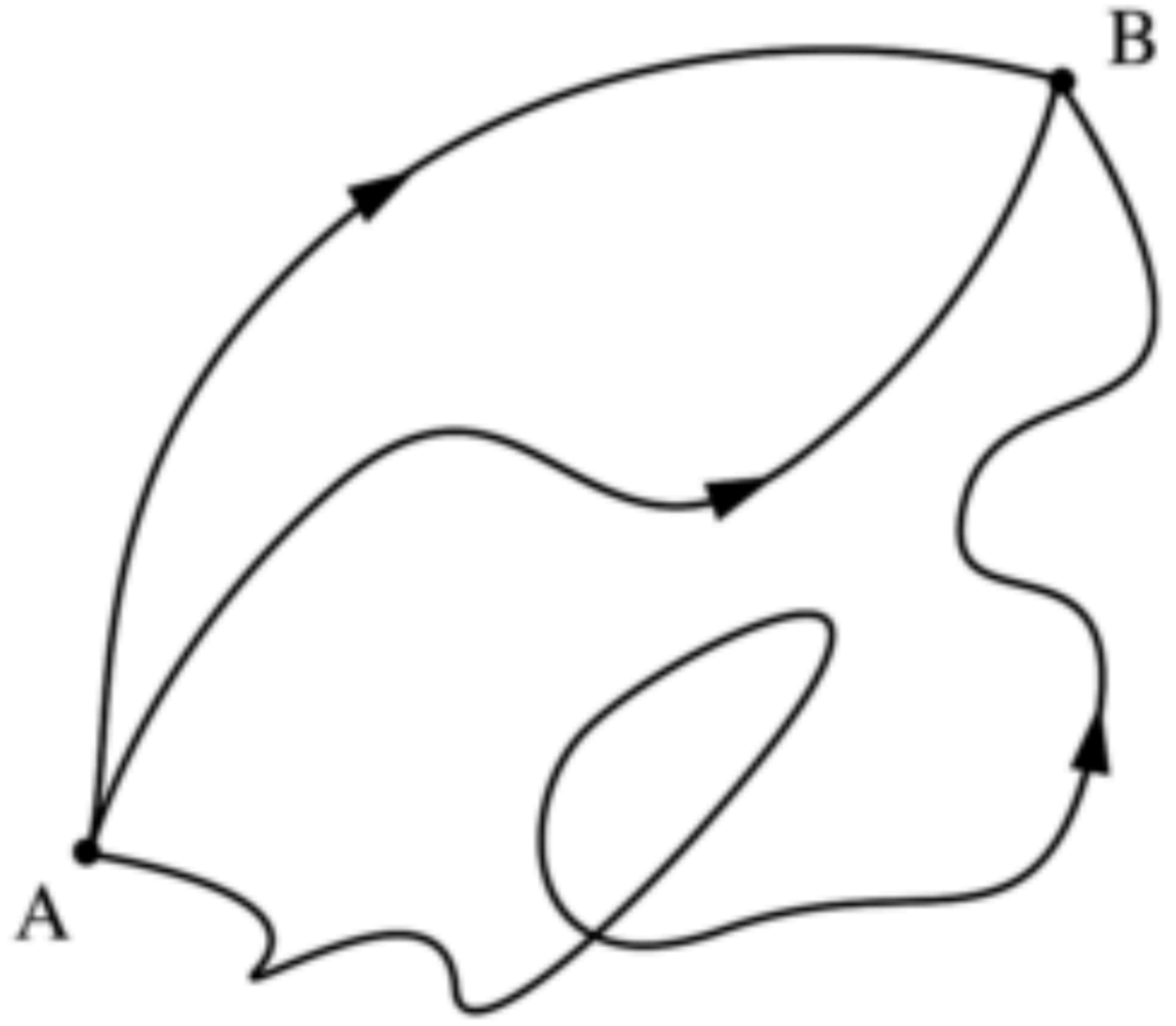


Trajectories of a system
between point A and B

How do I know the paths?

… and the one the nature "likes"?

Lagrangian



Trajectories of a system between point A and B

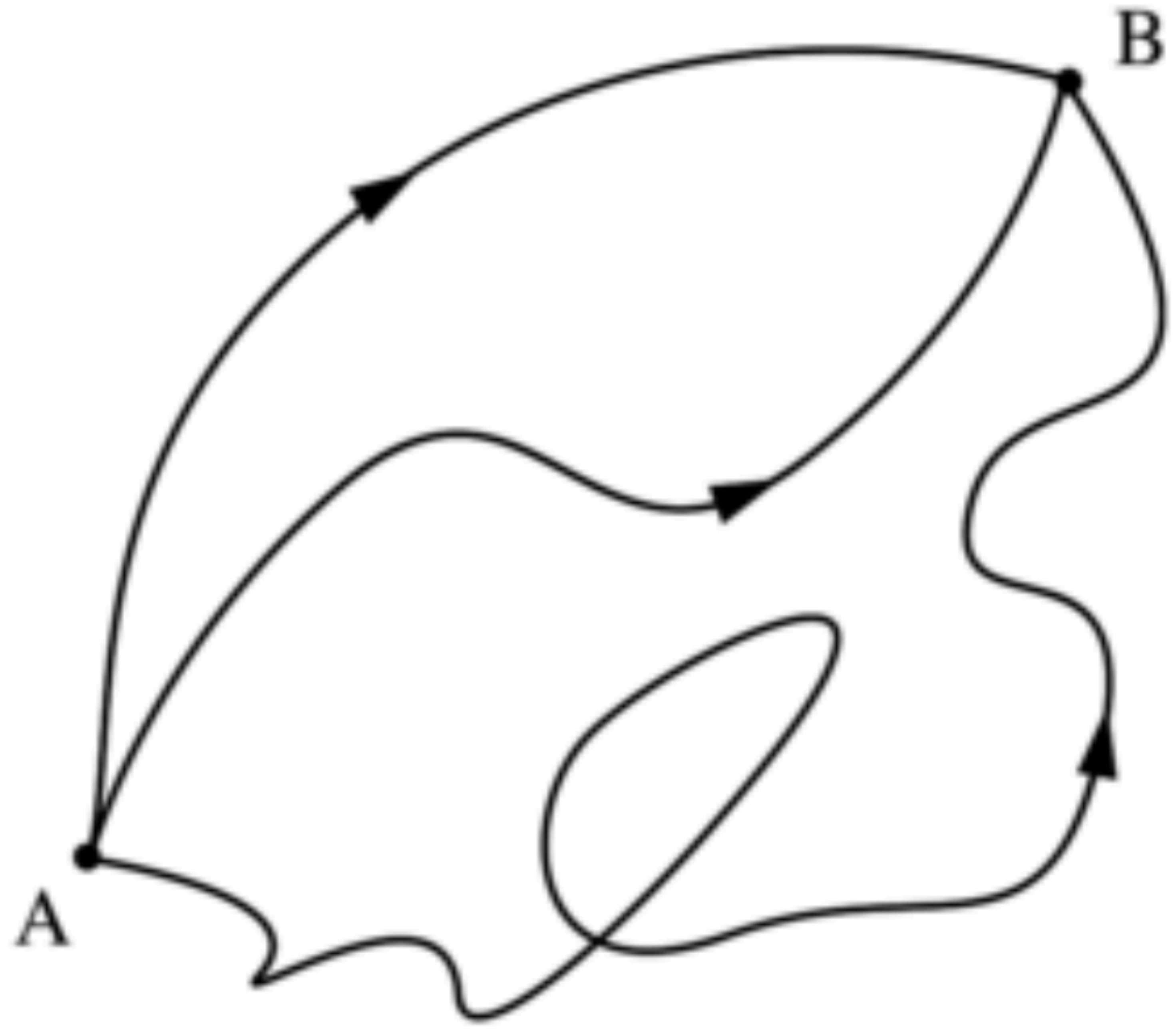
- There is a function, the **Lagrangian**, that contains all the information about the evolution of the system

$$\mathcal{L}(x(t), \dot{x}(t))$$

- Classically we can find it using,

$$\mathcal{L} = \mathrm{T} - \mathrm{V}$$

Euler-Lagrange equations



Trajectories of a system between point A and B

- The Euler-Lagrange equations are obtained by minimizing the action:

$$S = \int_{t_A}^{t_B} \mathcal{L}(x(t), \dot{x}(t))\, \mathrm{d}t$$

…nature likes to be efficient

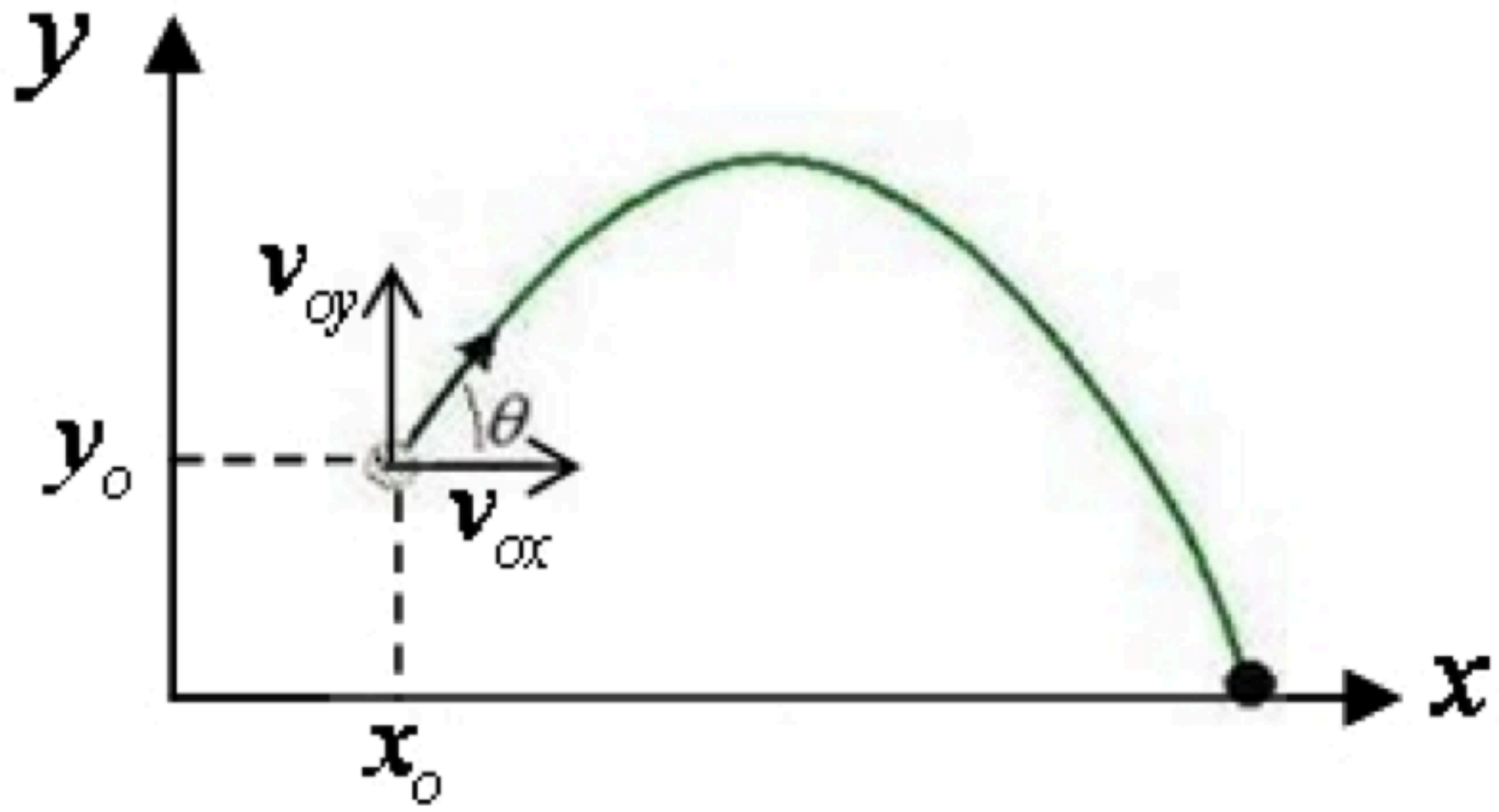
- The system trajectories satisfy:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0$$

$$\mathcal{L}(x(t), \dot{x}(t))$$

…for each coordinate

Example: Free particle in a gravitational field



$$\mathcal{L} = \mathrm{T} - \mathrm{V}$$

$$= \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgy$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0,$$

$$m\ddot{x} = 0 \quad \Rightarrow \quad \ddot{x} = 0$$

$$\Rightarrow x = b_1 t + b_2,$$

$$x(t) = x_o + v_{ox}t.$$

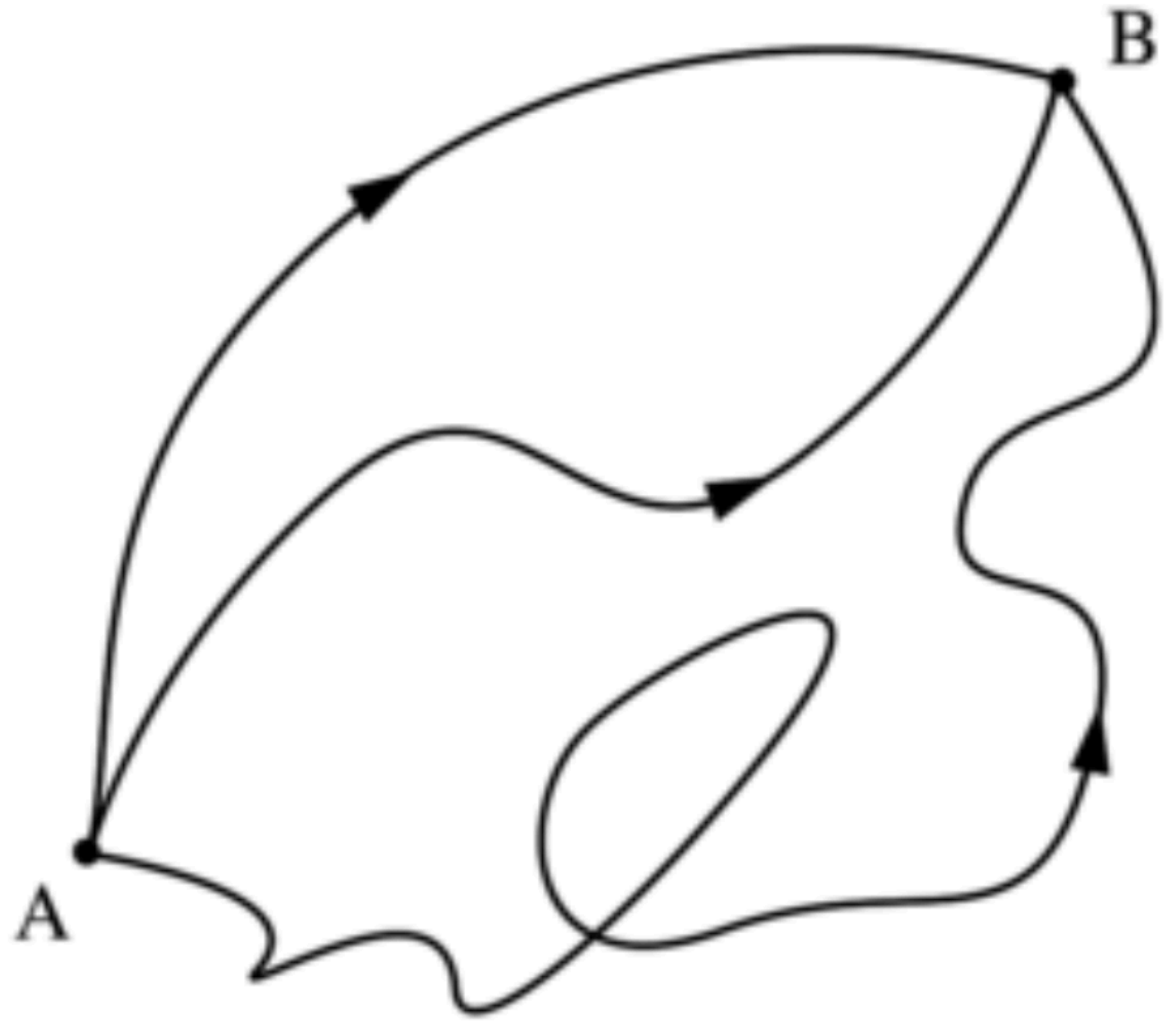
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = 0,$$

$$m\ddot{y} + mg = 0 \quad \Rightarrow \quad \ddot{y} = -g$$

$$\Rightarrow y = -\frac{1}{2}gt^2 + c_1 t + c_2$$

$$y(t) = y_o + v_{oy}t - \frac{1}{2}gt^2$$

Euler-Lagrange equations



Trajectories of a system between point A and B

- The Euler-Lagrange equations are obtained by minimizing the action:

$$S = \int_{t_A}^{t_B} \mathcal{L}(x(t), \dot{x}(t))\, \mathrm{d}t$$

$$x(t) \to \phi(x_\mu)$$

- The system trajectories satisfy:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0$$

$$\rightarrow \quad \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0.$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2 \phi^2$$

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\,\phi^2$$

$$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} - \frac{\partial\mathcal{L}}{\partial\phi} = 0$$

E6a:
Find the Euler-Lagrange equation

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\,\phi^2$$

$$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} - \frac{\partial\mathcal{L}}{\partial\phi} = 0$$

$$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = \partial_\mu\partial^\mu\,\phi$$

$$\frac{\partial\mathcal{L}}{\partial\phi} = m^2\phi$$

$$(\partial_\mu\partial^\mu - m^2)\phi = 0$$

$$(E^2 - P^2 - m^2)\psi = 0$$

$$E^2 = P^2 + m^2$$

Find the Euler-Lagrange equation of:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - J^{\mu}A_{\mu}$$

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} = -F_{\nu\mu}$$

$$= -\frac{1}{4} \left(\partial_{\mu}A_{\nu}\partial^{\mu}A^{\nu} - \partial_{\nu}A_{\mu}\partial^{\mu}A^{\nu} - \partial_{\mu}A_{\nu}\partial^{\nu}A^{\mu} + \partial_{\nu}A_{\mu}\partial^{\nu}A^{\mu}\right) - J^{\mu}A_{\mu}$$

$$= -\frac{1}{2} \left(\partial_{\mu}A_{\nu}\partial^{\mu}A^{\nu} - \partial_{\nu}A_{\mu}\partial^{\mu}A^{\nu}\right) - J^{\mu}A_{\mu}$$

$$\frac{\partial}{\partial(\partial_{\mu}A_{\nu})} \left(\partial_{\lambda}A_{\kappa}\partial^{\kappa}A^{\lambda}\right) = 2\partial^{\mu}A^{\nu}$$

$$\Rightarrow \quad -\partial_{\mu} \left(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}\right) + J^{\nu} = 0$$

$$\partial_{\mu}F^{\mu\nu} = \mu_0 J^{\nu}$$

E 6b: Find the DR of:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\partial_\mu F^{\mu\nu} = 0$$

$$\begin{aligned} \partial_\mu F^{\mu\nu} &= \partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) \\ &= \partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu \\ &= \partial_\mu \partial^\mu A^\nu \\ &= \left(\partial_0^2 - \nabla^2\right) A^\nu = 0 \end{aligned}$$

$$\Rightarrow \quad (w^2 - k^2)\tilde{A}^\nu = 0$$

E 6c: Find the DR of:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} \pm \epsilon F_{\mu\nu}F^{\mu\nu}$$

$$\alpha = \left(1 \pm \epsilon \right)$$

$$\partial_\mu \alpha = 0$$

$$\alpha \partial_\mu F^{\mu\nu} = \alpha\left(\partial_0^2 - \nabla^2\right)A^\nu = 0$$

$$\alpha(w^2 - k^2)\tilde{A}^\nu = 0$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \quad \Big| \quad (\partial_\mu\partial^\mu - m^2)\phi = 0 \quad \Big| \quad E^2 - p^2 = m^2$$

$$\mathcal{L} = \frac{1}{2}(1+\epsilon)\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2 \quad \Big| \quad (\alpha\partial_\mu\partial^\mu - m^2)\phi = 0 \quad \Big| \quad \alpha(E^2 - p^2) = m^2$$

$$\alpha = \left(1 \pm \epsilon \right)$$

$$\mathcal{L} = \frac{1}{2}(\alpha\partial_0\phi\partial^0\phi - \partial_i\phi\partial^i\phi - m^2\phi^2) \quad \Big| \quad (\alpha\partial_0\partial^0 - \partial_i\partial^i - m^2)\phi = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_0\phi\partial^0\phi - \alpha\partial_i\phi\partial^i\phi - m^2\phi^2) \quad \Big| \quad (\partial_0\partial^0 - \alpha\partial_i\partial^i - m^2)\phi = 0$$

$$\Big| \quad E^2 - p^2 \pm \epsilon A^2 = m^2,$$

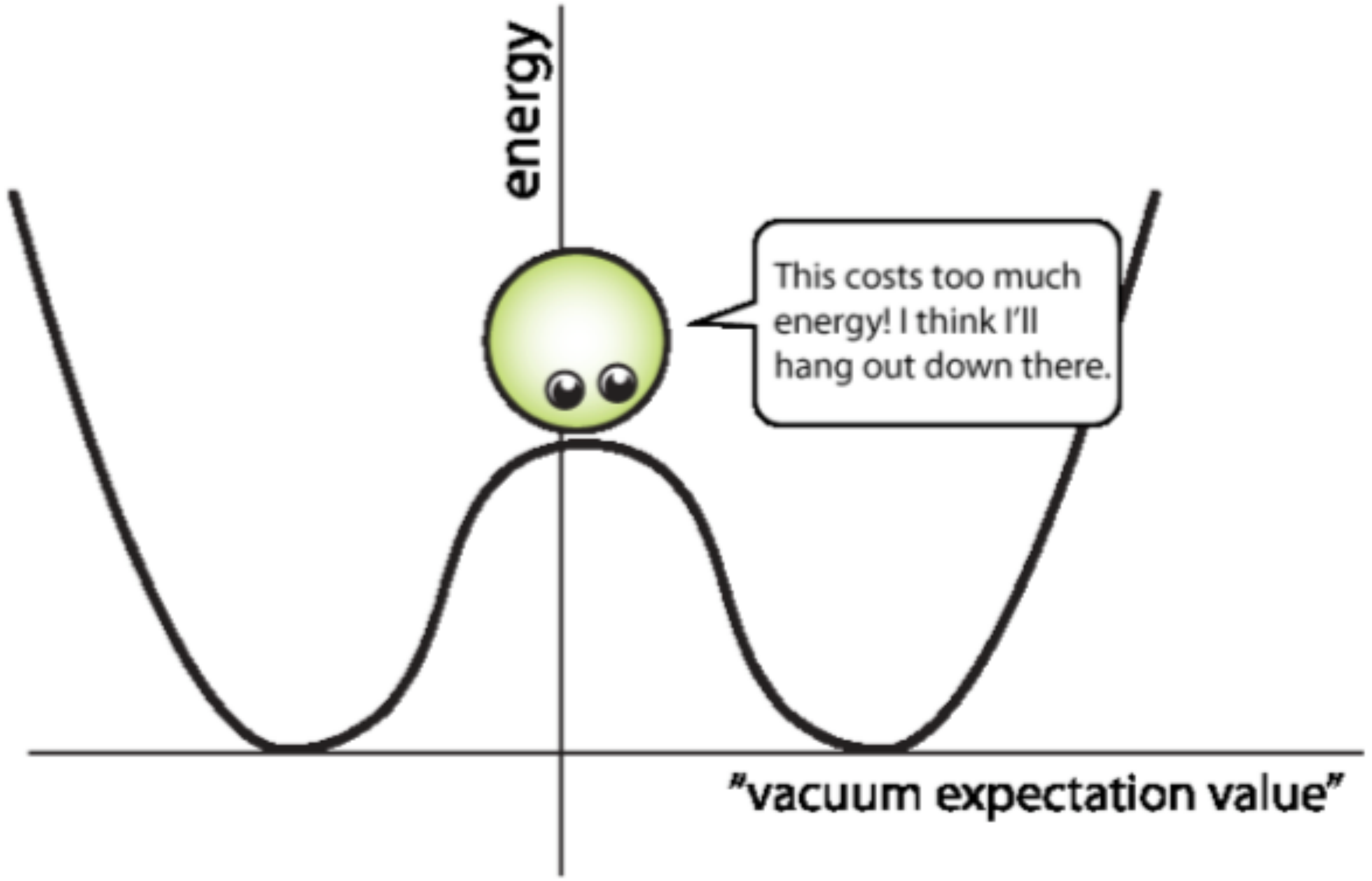
Spontaneous Symmetry Breaking

$$\mathcal{L} = \frac{1}{2}[(\partial\phi)^2 + \mu^2\phi^2] - \frac{\lambda}{4}(\phi^2)^2$$

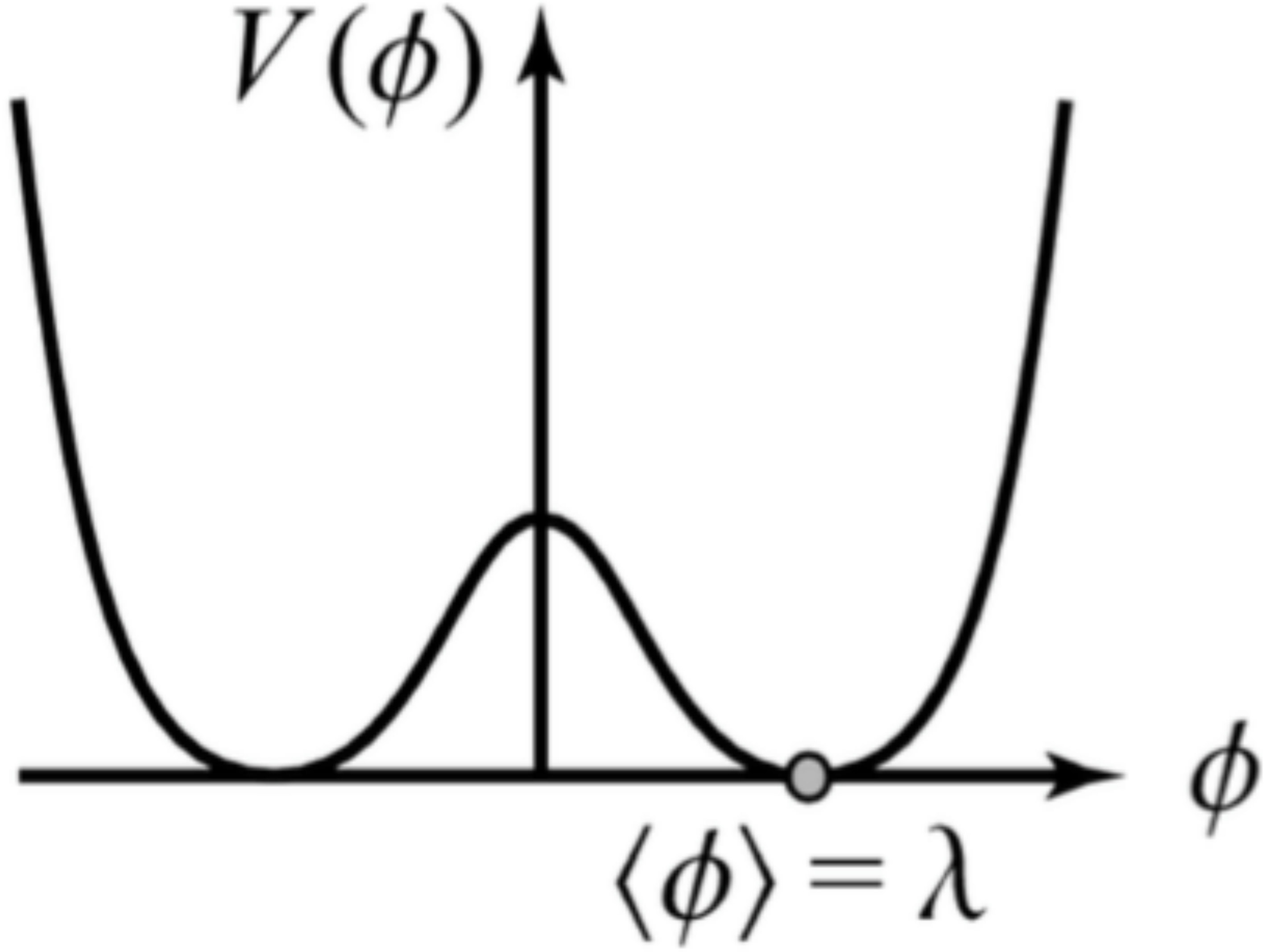
Two minima at $\phi = \pm v = \pm(\mu/\lambda)^{\frac{1}{2}}$.

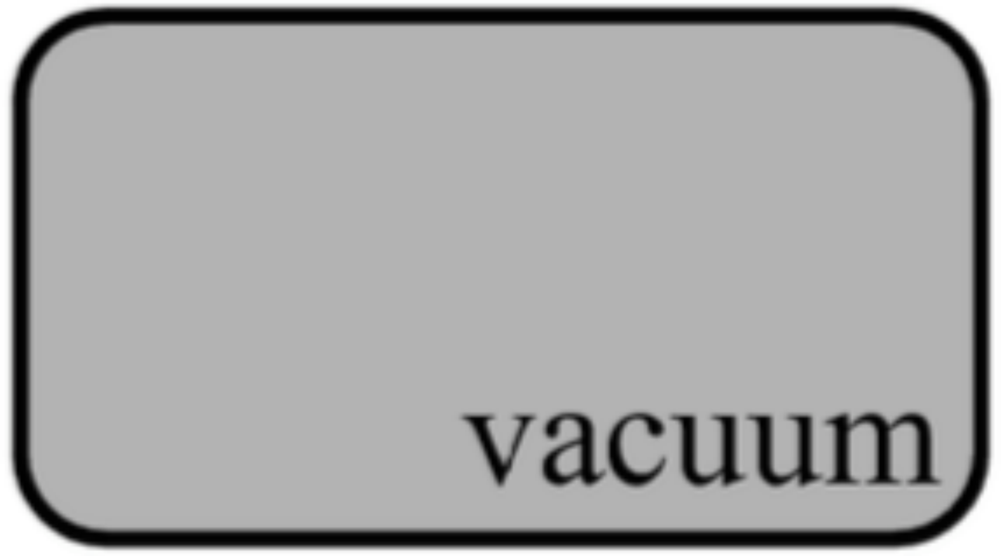
We have to commit to one or the other of the two possibilities for the ground state and build perturbation thoery around it.

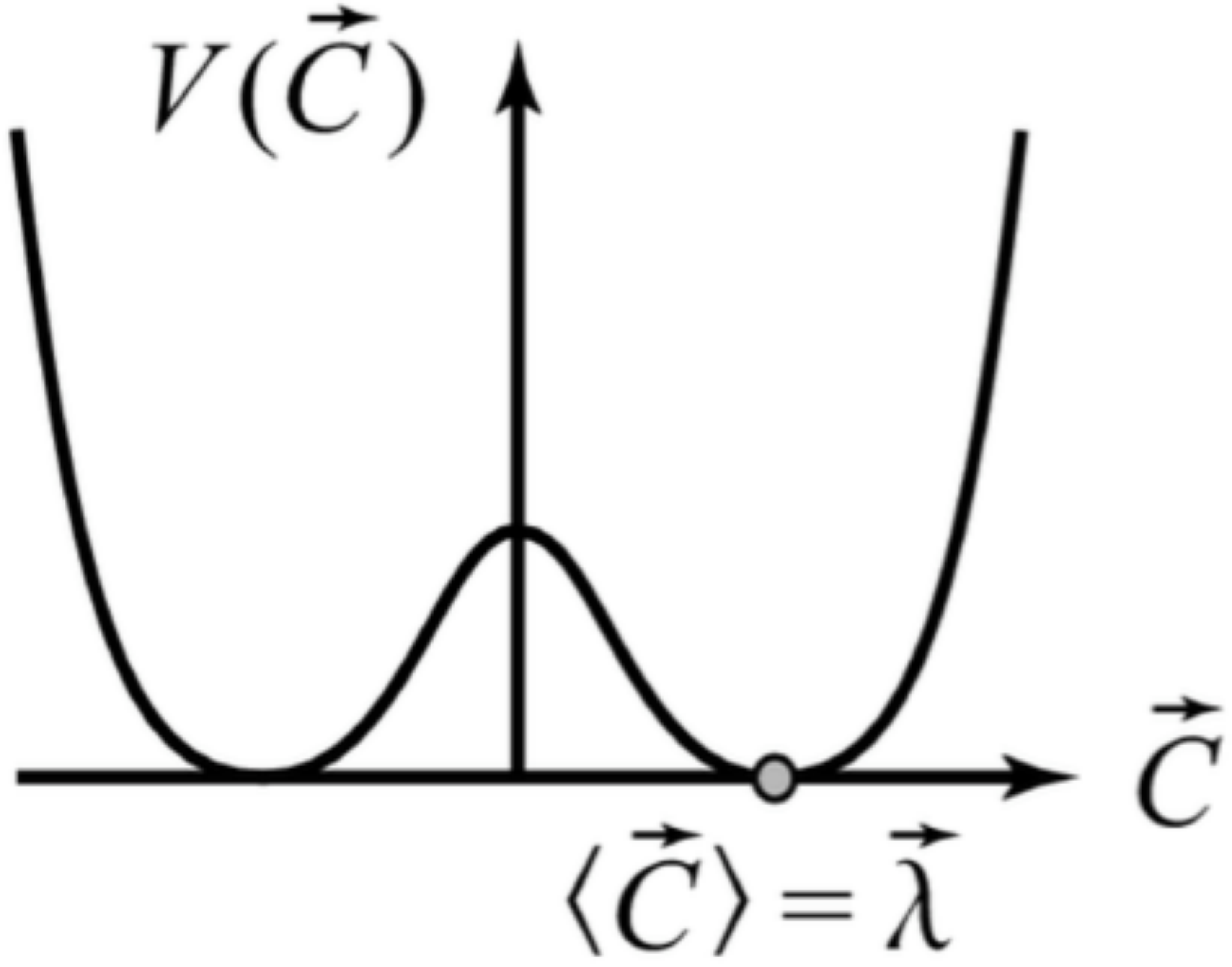
We did not put symmetry breaking terms into the Lagrangian by hand but yet the reflection symmetry is broken.

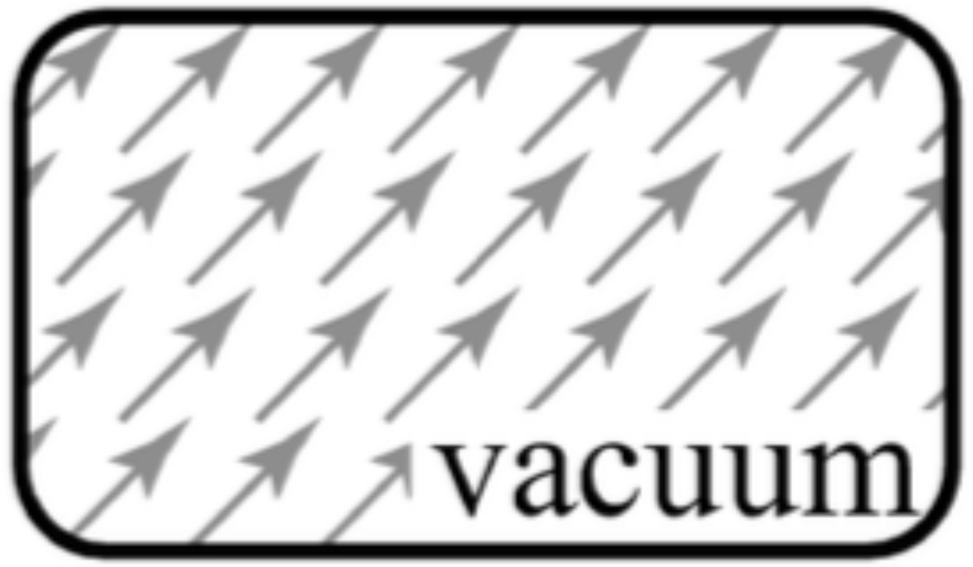


The reflection symmetry is broken spontaneously!


$V(\phi)$
ϕ
$\langle\phi\rangle = \lambda$


vacuum


$V(\vec{C})$
$\vec{C}$
$\langle\vec{C}\rangle = \vec{\lambda}$


vacuum

SM+GR+LIV : Effective QFT

Let $< T > \neq 0$

$$\mathcal{L} \approx \frac{\lambda}{m_P^k} < T > \Gamma \bar{\psi} (i\partial)^k \varphi \quad ; \quad \frac{\lambda}{m_P^k} < T > = t^{(k)}$$

$$\approx t^{(k)} \Gamma \bar{\psi} (i\partial)^k \varphi \quad ;$$

Tensor valued backgrounds.
Preferred direction in spacetimes
-> they introduce Lorentz Violation.

Many LIV-models:

- Maintain spatial rotation invariance while breaking boost.
- Bumblebee model
- LV dispersion relation.
- Vector tensor Models in gravity that SB LS
- …

EFT-QFT $\longrightarrow$ SME

Spacetime Symmetries in Relativity

PP

Internal Gauge Symmetries: $SU(3) \times SU(2) \times U(1)$

Discrete spacetime symmetries: C, P, T

Many are broken

Explicitly:
- weak interactions,
- certain meson interactions

SSB:
- Higgs mechanism: EW-M

Global Lorentz Symmetry!

s-t symmetry : SR

But CPT is conserved

(for local interactions of point like particles in QFT)

*Any Exp. looking for **CTP-V** -> **LS-V** test (in QFT)

Spacetime Symmetries in Relativity

GR

Effects of gravity
by
curved s-t

$g_{\mu\nu}$ The metric tensor

$R^{\kappa}_{\lambda\mu\nu}$ Riemann Curvature Tensor

$R_{\mu\nu}$ Ricci

R Curvature

$T^{M}_{\mu\nu}$ Source of the s-t curvature

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

↓

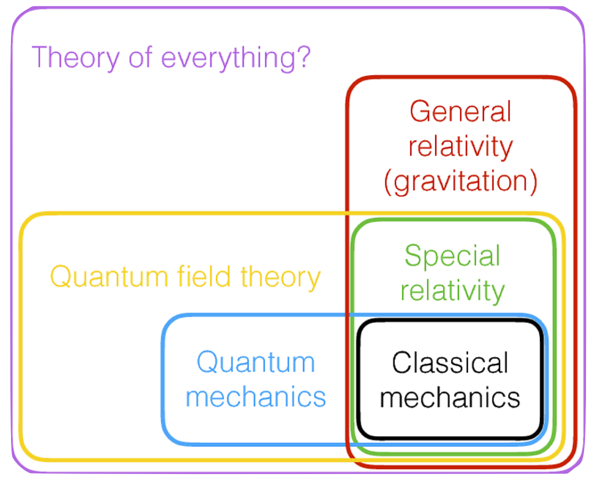
Invariant under Diffeomorphisms

Local Lorentz Invariant

Lorentz Invariance Violation

PP + GR

LIV



- SM & GR: the best theories describing the 4-fundamental Forces.
- No conflict with predictions from either of them.
- **They are fundamentally different.**

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{GR}$$

$$+\mathcal{L}_{LIV}$$

These terms consist of Lorentz-violating operators of mass dimension three or four, coupled to coefficients with Lorentz indices controlling the degree of Lorentz violation. The subset of the theory containing these dominant Lorentz-violating terms is called the minimal SME

Searches for violations can take advantage either of gravitational or of nongravitational forces, or of both.

$$\mathcal{L}_{SM} = \mathcal{L}_{LI} + \mathcal{L}_{LIV}$$

$$\mathcal{L}_{G} = \mathcal{L}_{LI} + \mathcal{L}_{LIV}$$

D. Colladay and V. A. Kostelecky,
Phys. Rev. D 58, 116002 (1998).

M. Schreck
Phys.Rev. D96 (2017) no.9, 095026

V. Alan Kostelecky and Neil Russell
https://arxiv.org/pdf/0801.0287.pdf

A. Bourgoin et al
https://arxiv.org/abs/1706.06294v3

Q. G. Bailey and V. A. Kostelecky,
Phys. Rev. D 74, 045001 (2006).

V.A.Kostelecky andJ.D.Tasson,
Phys.Rev.D83, 016013 (2011)

Since the SME is founded on well established physics and constructed from general operators, it offers an approach to describing Lorentz violation that is largely independent of the underlying theory.

SM + LIV

$$L_A = \begin{pmatrix} \nu_A \\ l_A \end{pmatrix}_L, \quad R_A = (l_A)_R, \quad Q_A = \begin{pmatrix} u_A \\ d_A \end{pmatrix}_L, \quad U_A = (u_A)_R, \quad D_A = (d_A)_R,$$

A= 1, 2, 3 labels the flavor: $l_A = (e, \mu, \tau)$, $\nu_A = (\nu_e, \nu_\mu, \nu_\tau)$, $u_A = (u, c, t)$, $d_A = (d, s, b)$.

The Higgs doublet: $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ r_\phi \end{pmatrix}$; the Gauge Fields: G_μ, W_μ and $G_{\mu\nu}$

$$\mathcal{L}_{lepton} = \frac{1}{2} i \bar{L}_A \gamma^\mu \overleftrightarrow{D}_\mu L_A + \frac{1}{2} i \bar{R}_A \gamma^\mu \overleftrightarrow{D}_\mu R_A + \ldots$$

$$\mathcal{L}_{quark} = \frac{1}{2} i \bar{Q}_A \gamma^\mu \overleftrightarrow{D}_\mu Q_A + \frac{1}{2} i \bar{U}_A \gamma^\mu \overleftrightarrow{D}_\mu U_A + \frac{1}{2} i \bar{D}_A \gamma^\mu \overleftrightarrow{D}_\mu D_A + \ldots$$

$$\mathcal{L}_{Yukawa} = -[(G_L)_{AB} \bar{L}_A \phi R_B + (G_U)_{AB} \bar{L}_A \phi^c R_B + (G_D)_{AB} \bar{Q}_A \phi D_B] + H.C. + \ldots$$

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{3!} (\phi^\dagger \phi)^2 + \ldots$$

$$\mathcal{L}_{Gauge} = -\frac{1}{2} Tr(G_{\mu\nu} G^{\mu\nu}) - \frac{1}{2} Tr(W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \ldots$$

$$\mathcal{L}^{\text{CPT-even}}_{\text{lepton}} = \tfrac{1}{2} i (c_L)_{\mu\nu AB} \overline{L}_A \gamma^\mu \overleftrightarrow{D^\nu} L_B + \tfrac{1}{2} i (c_R)_{\mu\nu AB} \overline{R}_A \gamma^\mu \overleftrightarrow{D^\nu} R_B$$

$$\mathcal{L}^{\text{CPT-odd}}_{\text{lepton}} = -(a_L)_{\mu AB} \overline{L}_A \gamma^\mu L_B - (a_R)_{\mu AB} \overline{R}_A \gamma^\mu R_B$$

$$\mathcal{L}^{\text{CPT-even}}_{\text{gauge}} = -\tfrac{1}{2} (k_G)_{\kappa\lambda\mu\nu} \text{Tr}(G^{\kappa\lambda} G^{\mu\nu}) - \tfrac{1}{2} (k_W)_{\kappa\lambda\mu\nu} \text{Tr}(W^{\kappa\lambda} W^{\mu\nu}) - \tfrac{1}{4} (k_B)_{\kappa\lambda\mu\nu} B^{\kappa\lambda} B^{\mu\nu} \quad .$$

coefficients $k_{G,W,B}$ are real.

$$\mathcal{L}^{\text{CPT-odd}}_{\text{gauge}} = (k_3)_\kappa \epsilon^{\kappa\lambda\mu\nu} \text{Tr}(G_\lambda G_{\mu\nu} + \tfrac{2}{3} i g_3 G_\lambda G_\mu G_\nu) + (k_2)_\kappa \epsilon^{\kappa\lambda\mu\nu} \text{Tr}(W_\lambda W_{\mu\nu} + \tfrac{2}{3} i g W_\lambda W_\mu W_\nu) + (k_1)_\kappa \epsilon^{\kappa\lambda\mu\nu} B_\lambda B_{\mu\nu} + (k_0)_\kappa B^\kappa$$

k1,2,3 are real and have dimensions of mass,; k0 is also real and has dimensions of mass 3

THE PURE-PHOTON SECTOR

$$\mathcal{L}^{\text{total}}_{\text{photon}} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{4} (k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu} + \tfrac{1}{2} (k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} A^\lambda F^{\mu\nu} \quad .$$

D. Colladay and V. A. Kostelecky, Phys. Rev. D 58, 116002 (1998).

Standard Model Extension d=4 (n=0)

Isotropic Lorentz- violating (LV) deformation of the photon sector

$$\mathcal{L}_{modM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\kappa^{\mu\nu}{}_{\rho\lambda}F_{\mu\nu}F^{\rho\lambda}.$$

D. Colladay and V.A. Kostelecký,
Phys. Rev. D 58, 116002 (1998)

Kappa:

$$\kappa^{\mu\nu}{}_{\mu\nu} = 0 \quad ; \qquad \kappa_{\mu\nu\rho\lambda} = -\kappa_{\nu\mu\rho\lambda} = \kappa_{\nu\mu\lambda\rho}, \quad \kappa_{\mu\nu\rho\lambda} = \kappa_{\rho\lambda\mu\nu}$$

256 independent components to 19

$$\kappa^{\mu\nu\rho\lambda} = \frac{1}{2}(\eta^{\mu\rho}\tilde{\kappa}^{\nu\lambda} - \eta^{\mu\lambda}\tilde{\kappa}^{\nu\rho} + \eta^{\nu\lambda}\tilde{\kappa}^{\mu\rho} - \eta^{\nu\rho}\tilde{\kappa}^{\mu\lambda}) \; ; \quad \tilde{\kappa}^{\mu\nu} = \frac{3}{2}\tilde{\kappa}_{tr}\ \mathrm{diag}(1, 1/3, 1/3, 1/3)$$

19 independent components to 1

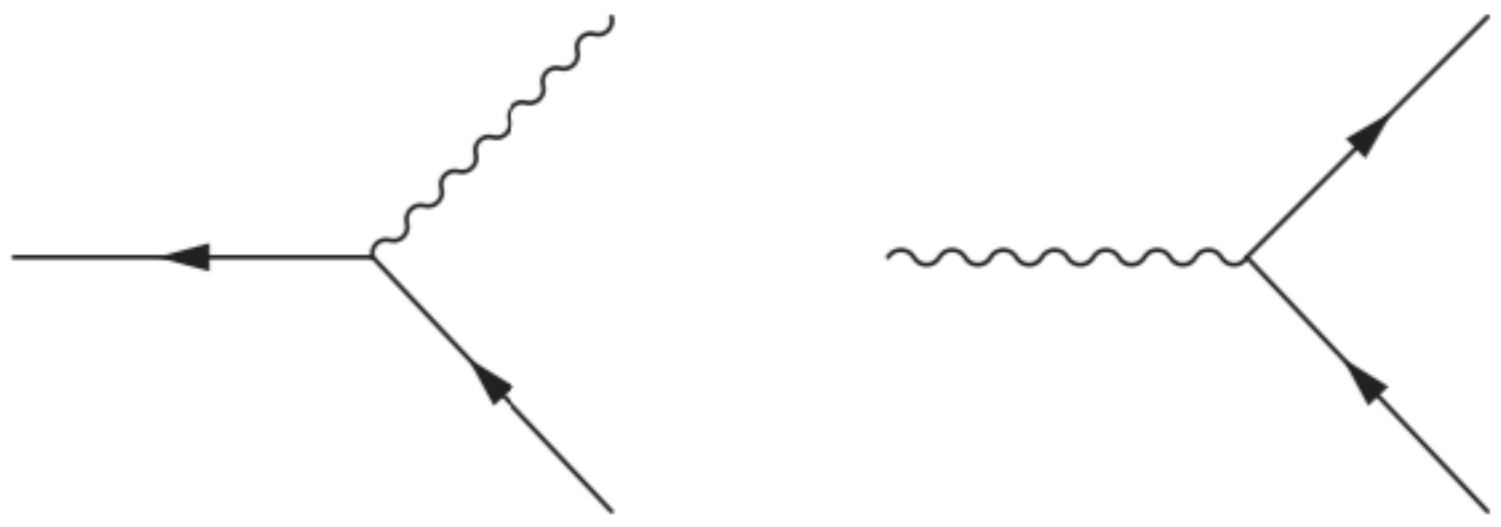
F. R. Klinkhamer and M. Schreck
Phys. Rev. D **78**, 085026 (2008)

M. Hohensee, R. Lehnert, D. Phillips, R. Walsworth
Phys. Rev. D **80**, 036010 (2009)

LORENTZ-NONINVARIANT DECAY PROCESSES

a) Vacuum Cherenkov radiation

b) Photon decay



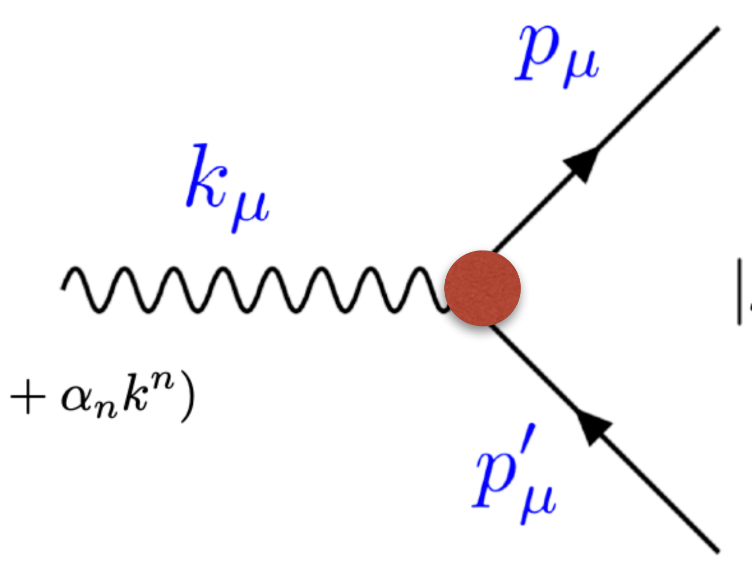
(a) (b)

Photon decay Threshold

$$-2\tilde{\kappa}_{tr} \sim 4m_e^2/E_\gamma^2$$

$$\alpha_0 = -2\tilde{\kappa}_{tr}$$

Generic framework: Photon decay

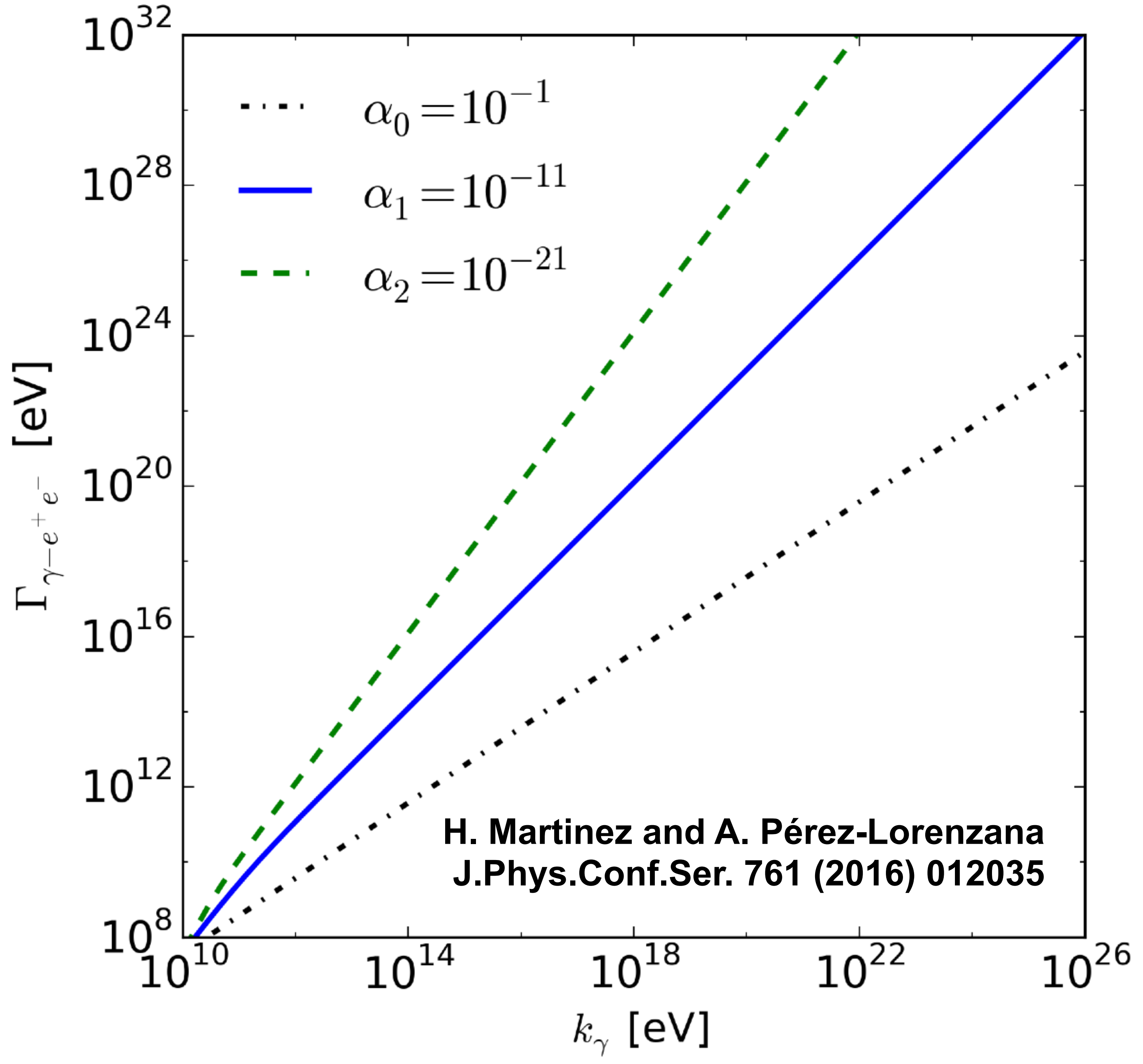


$\omega^2 = k^2(1+\alpha_n k^n)$

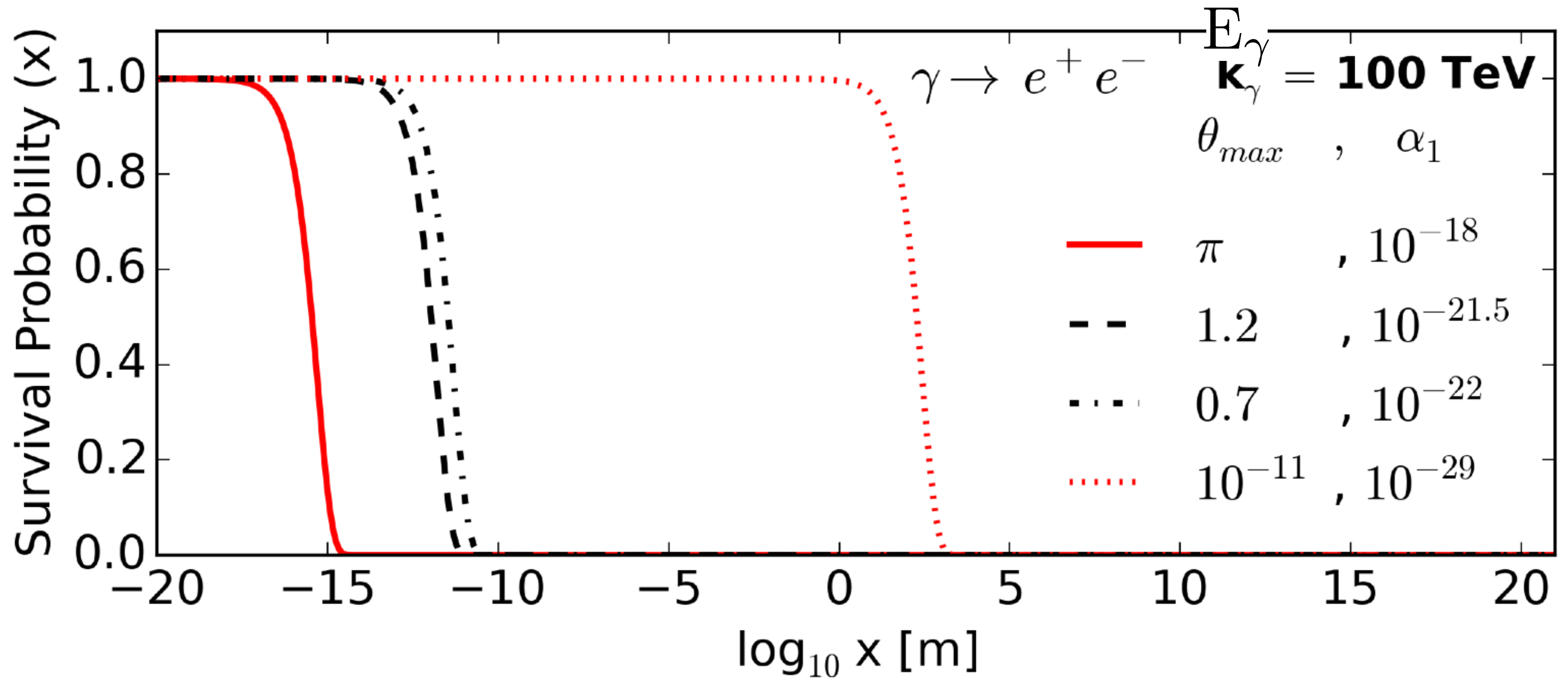
$|\mathcal{M}|^2 = e^2|4m^2 - \alpha_n k^{n+2}|,$

$$\alpha_{\gamma,n} < E_\gamma^{-n}\left[\frac{4m_e^2}{E_\gamma^2 - 4m_e^2}\right]$$

decay rate



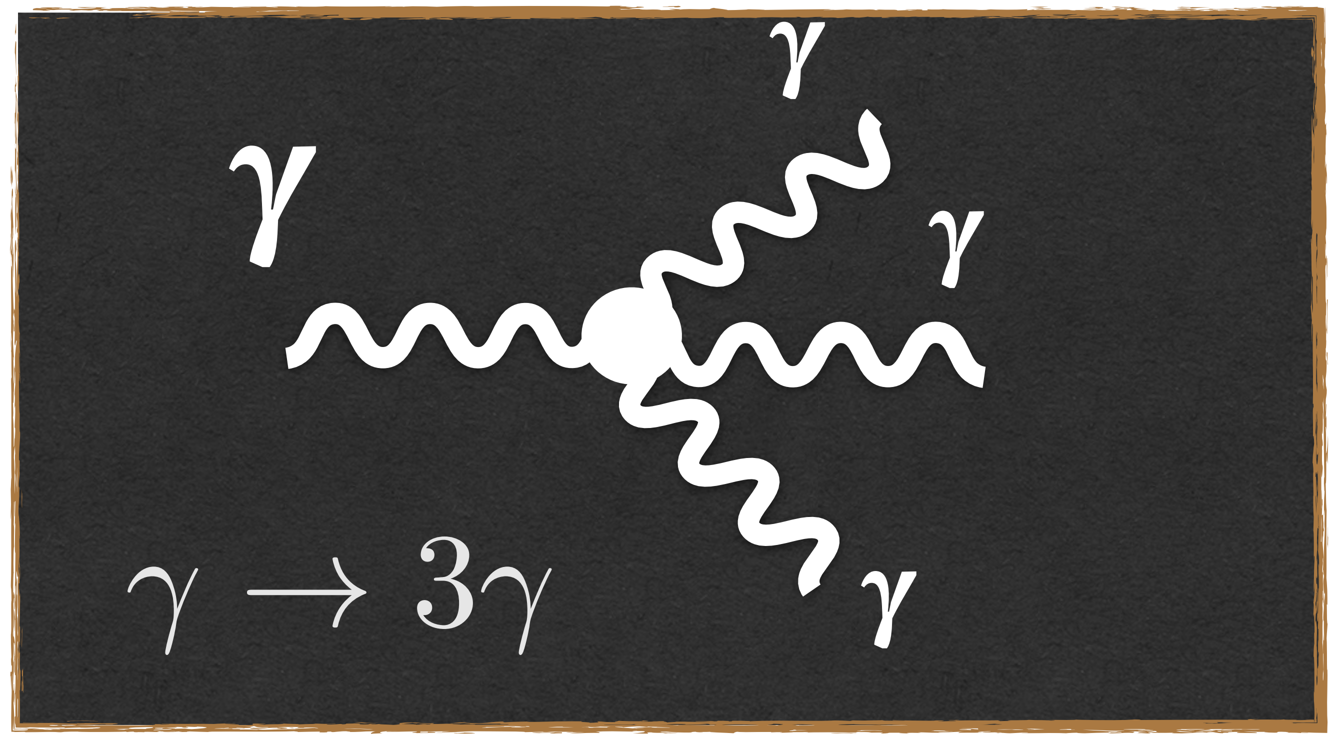
Above this energy threshold, the decay rate is quite efficient that photons should not arrive at Earth from cosmological distances



If you observe VHE gamma-rays, the LIV process is restricted!!

H. Martinez and A. Lorenzana
Phys.Rev. D95 (2017) 6, 063001

Photon splitting



$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2M_{LV}^2}F_{ij}\Delta^2 F^{ij}$$

$$E_\gamma^2 = k_\gamma^2 + \frac{k_\gamma^4}{M_{LV}^2}.$$

n=2

$$\Gamma_{\gamma\to 3\gamma} = 5\times 10^{-14}\frac{{E_\gamma}^{19}}{{m_e}^8 E_{LIV}^{(2)\ 10}},$$

no threshold!

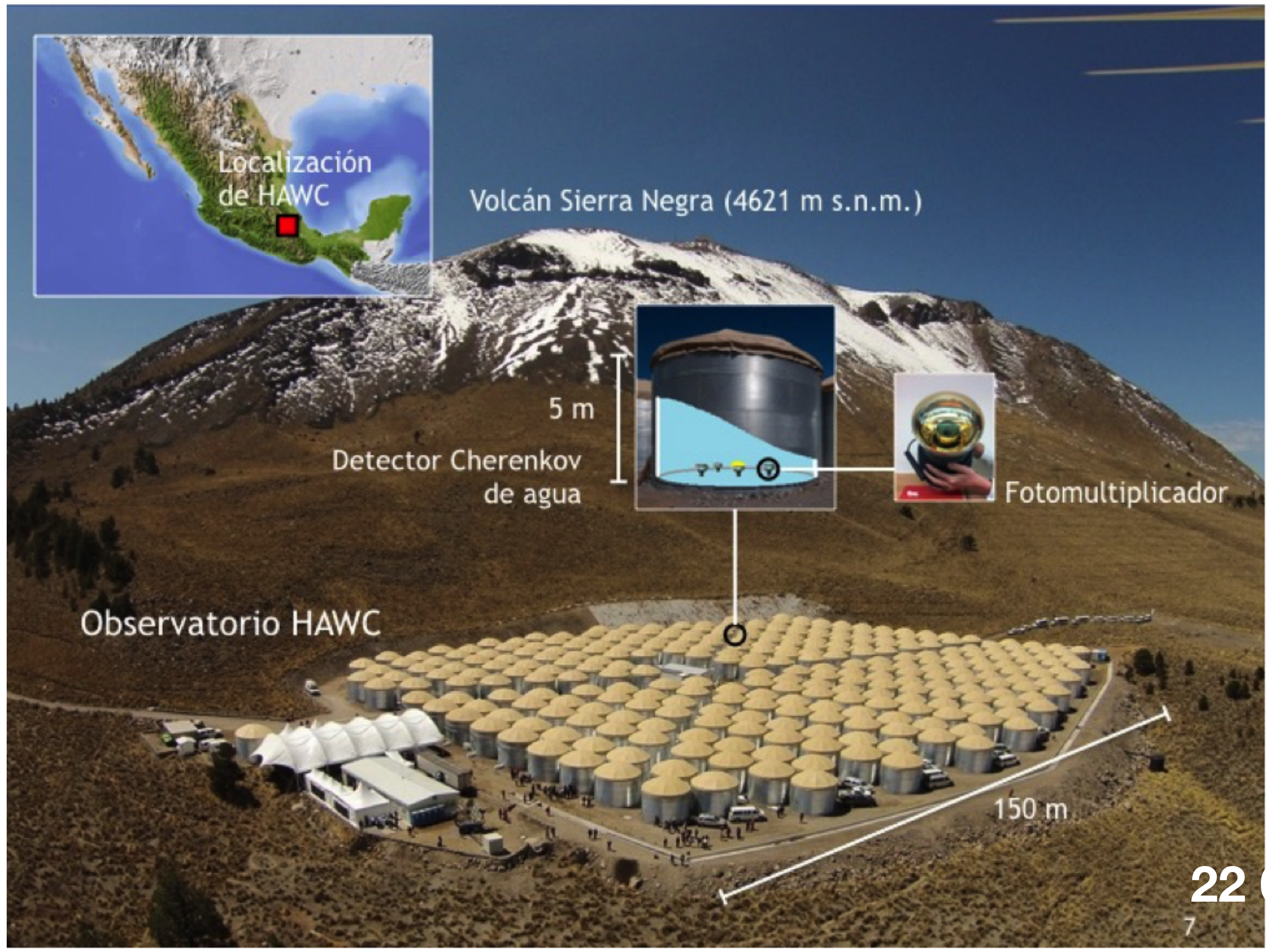
It becomes significant when photons propagate through cosmological distances -> cutoff

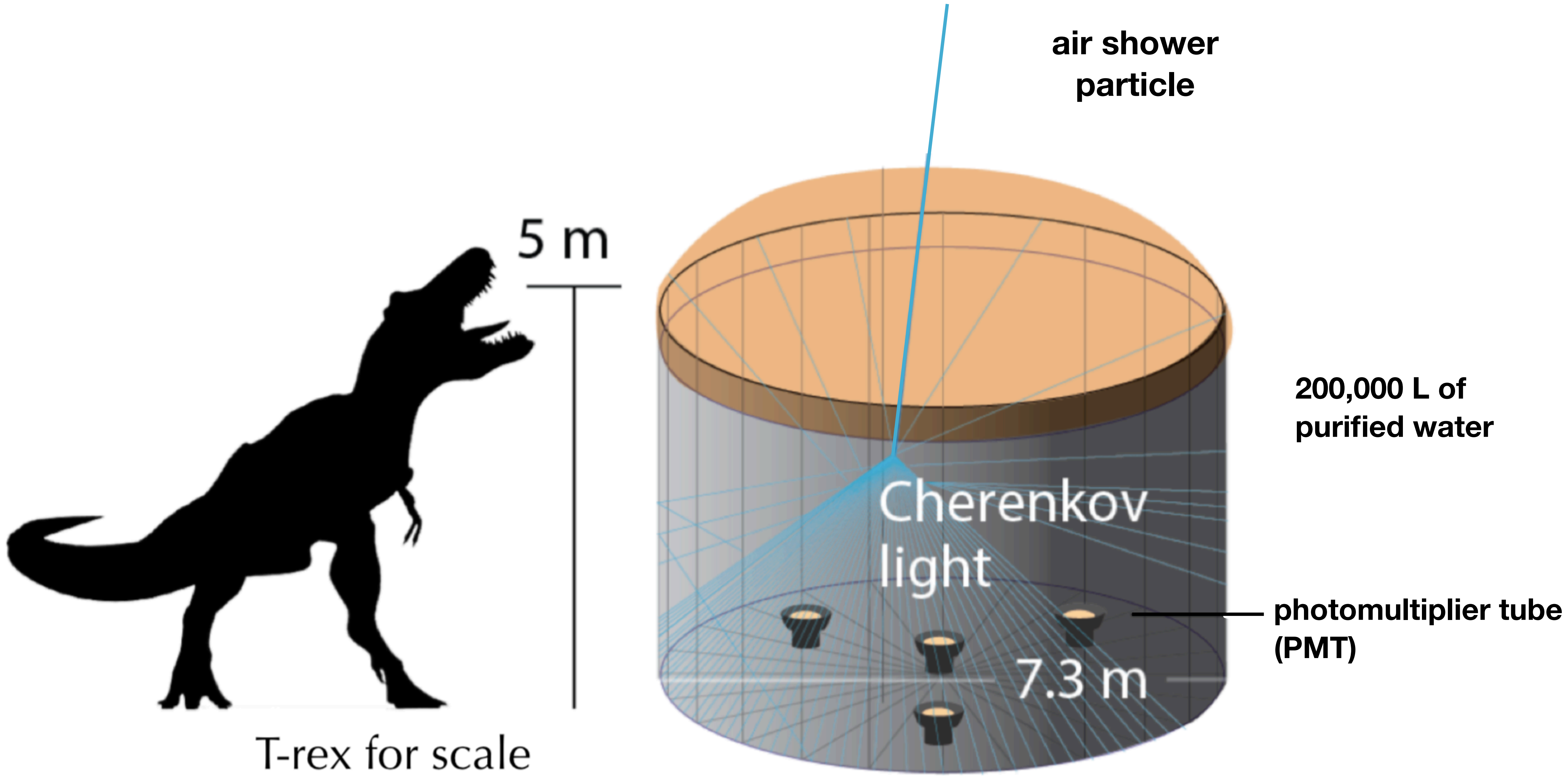
$$L\,\Gamma = 1$$

If you observe VHE gamma-rays,
the LIV process is restricted!!

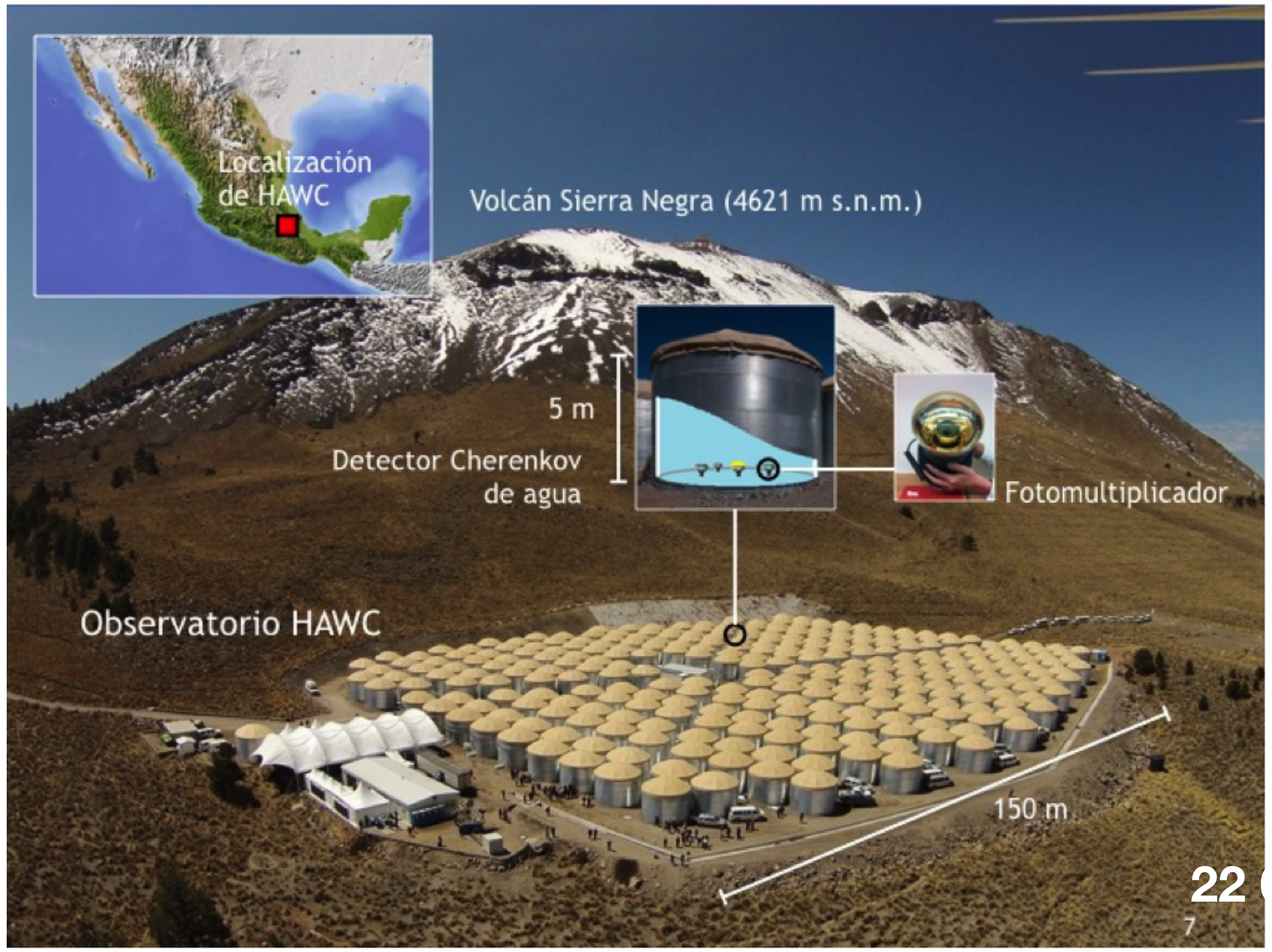
K. Astapov, D. Kirpichnikov, and P. Satunin, JCAP 1904, 054 (2019)

The High Altitude Water Cherenkov

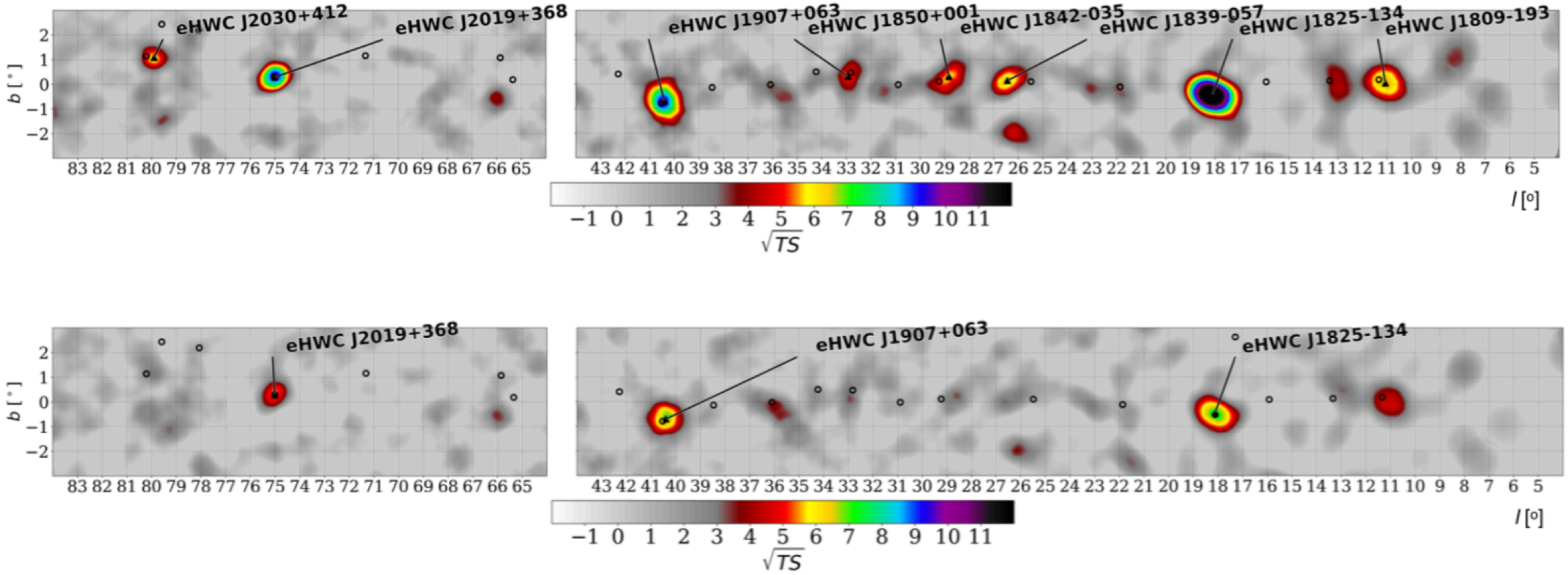




The High Altitude Water Cherenkov



Highest energy sources



> 56 TeV:

> 100 TeV:

-Reported detailed measurements of γ-ray >**100 TeV**,

-Recent development of advanced energy-reconstruction algorithms, **artificial neural network**

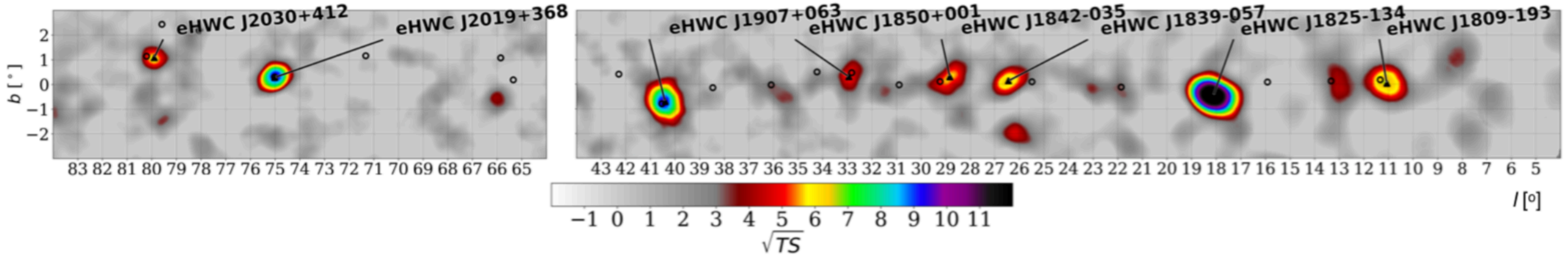
- ☑ **Crab**,
- ☑ 2HWC J1825−134,
- ☑ 2HWC J1907+063,
- ☑ 2HWC J2019+368

HAWC Collaboration

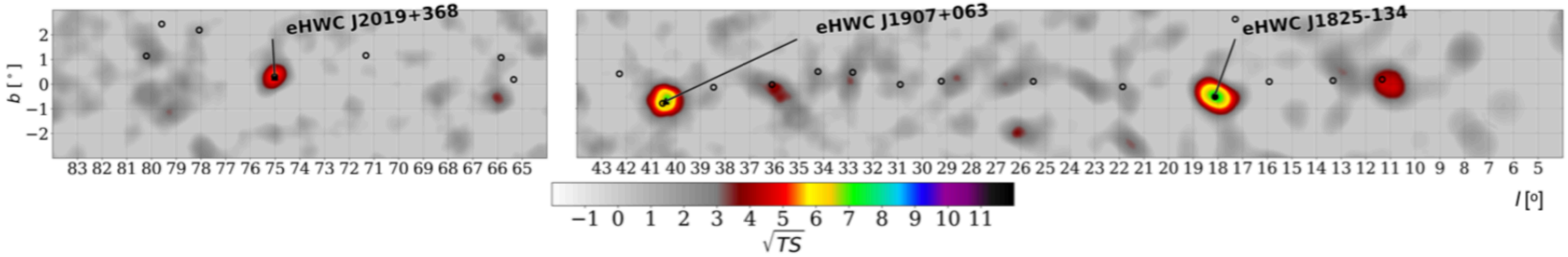
Phys Rev Lett. 124, 021102 (2020)

Highest energy sources

HAWC High Altitude Water Cherenkov Gamma-Ray Observatory



> 56 TeV:



> 100 TeV:

☑ **Crab**,
☑ 2HWC J1825−134,
☑ 2HWC J1907+063,
☑ 2HWC J2019+368

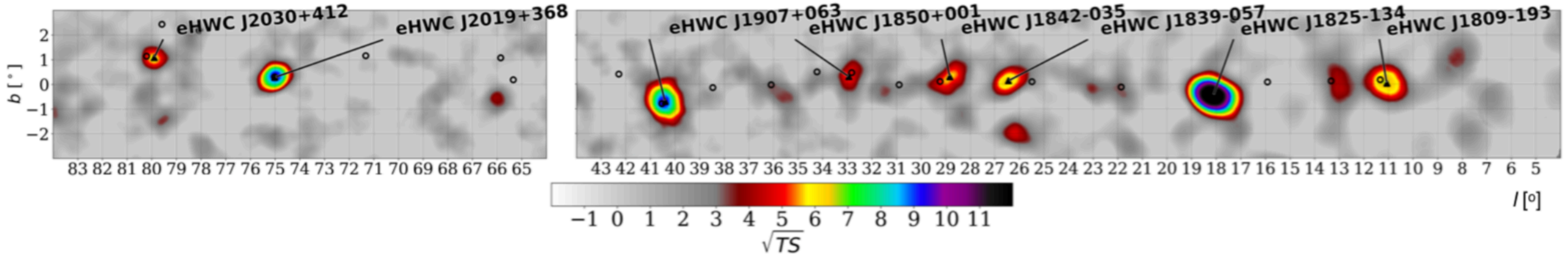
HAWC Collaboration
Phys Rev Lett. 124, 021102 (2020)

Above this energy threshold, the decay rate is quite efficient that photons should not arrive at Earth from cosmological distances

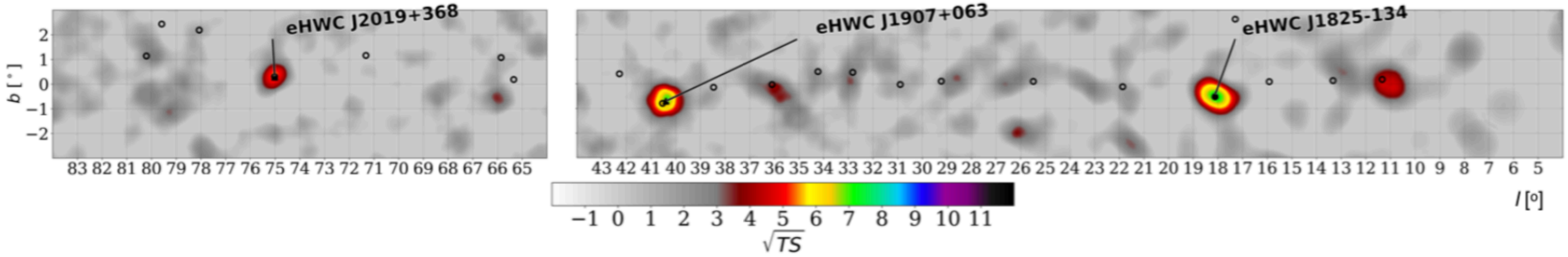
$$E_{LIV}^{(n)} > E_\gamma \left[\frac{E_\gamma^2 - 4m_{e^-}^2}{4m_{e^-}^2} \right]^{1/n}$$

Highest energy sources

High Altitude Water Cherenkov
Gamma-Ray Observatory



> 56 TeV:



> 100 TeV:

- ☑ **Crab**,
- ☑ 2HWC J1825−134,
- ☑ 2HWC J1907+063,
- ☑ 2HWC J2019+368

HAWC Collaboration
Phys Rev Lett. 124, 021102 (2020)

Above this energy threshold, the decay rate is quite efficient that photons should not arrive at Earth from cosmological distances

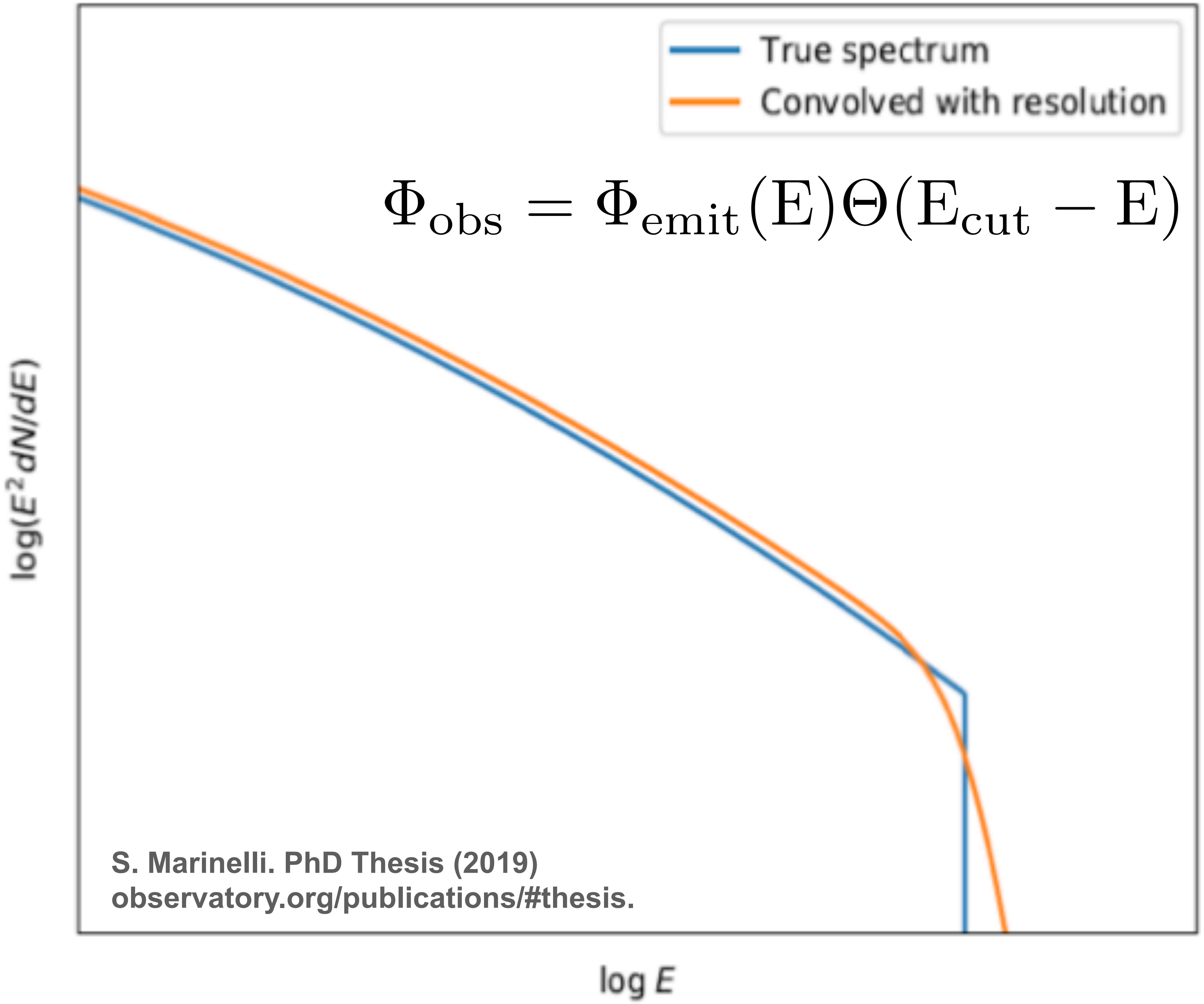
LIV hard cutoff

LIV hard cutoff at some energy Ec in the True spectrum

softened in the observed spectra due to the effects of the detector energy resolution

A profile log-likelihood is performed to find the best-fit spectrum model for each source, including a energy cutoff, $\hat{E}_c$

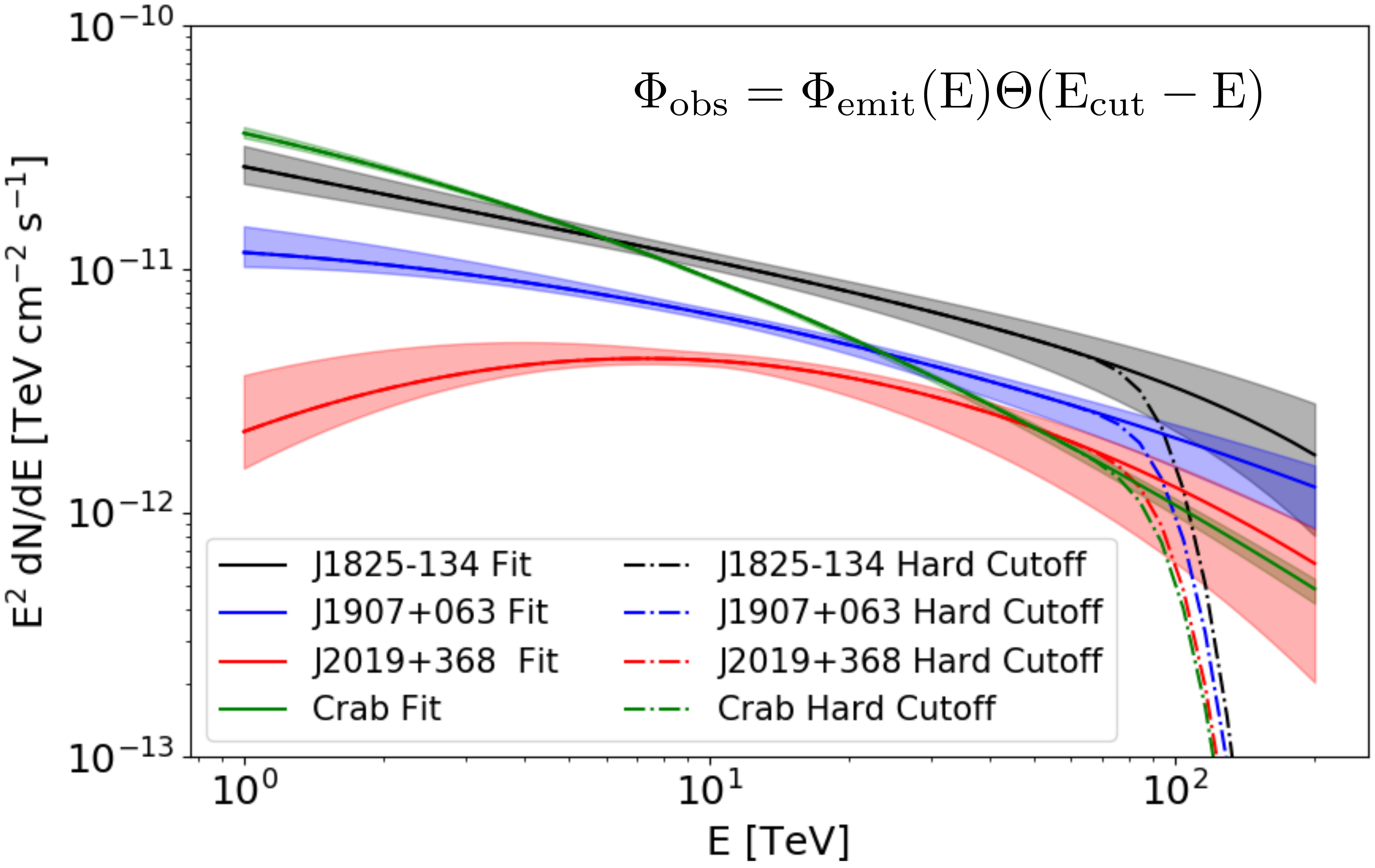
$$D = 2\ln\left(\frac{\mathscr{L}(\hat{E}_c)}{\mathscr{L}(\hat{E}_c \to \infty)}\right)$$



$$\Phi_{obs} = \Phi_{emit}(E)\Theta(E_{cut} - E)$$

S. Marinelli. PhD Thesis (2019)
observatory.org/publications/#thesis.

LIV hard cutoff



Source	p-value	$E_c(95\%)$	$E_c(3\sigma)$
eHWC J1825-134	1.000	244	158
eHWC J1907+063	0.990	218	162
eHWC J0534+220 (Crab)	1.000	152	104
eHWC J2019+368	0.828	120	88

Table I. HAWC sources and Photon Energy Limits (TeV).

Small p value
data favors a cutoff

No statistically significant
evidence of **hard cutoffs**

HAWC Collaboration
Phys Rev Lett. 124,131101 (2020)

LIV limits

Above this **energy threshold,** the decay rate is quite efficient that photons should not arrive at Earth from cosmological distances

$$E_{LIV}^{(n)} > E_\gamma \left[\frac{E_\gamma^2 - 4m_{e^-}^2}{4m_{e^-}^2} \right]^{1/n}$$

Source	E_c TeV	L kpc	α_0 10^{-17}	α_1 $10^{-32}\mathrm{eV}^{-1}$	α_2 $10^{-48}\mathrm{eV}^{-2}$	$\alpha_{2(3\gamma)}$ $10^{-48}\mathrm{eV}^{-2}$	$E_{LIV}^{(1)}$ 10^{31}eV	$E_{LIV}^{(2)}$ 10^{23}eV
eHWC J1825-134	244	1.55	1.75	7.19	295	0.70	1.39	0.58
eHWC J1907+063	218	2.37	2.2	10.1	462	0.99	0.99	0.47
eHWC J0534+220 (Crab)	152	2	4.52	29.7	1960	4.01	0.34	0.23
eHWC J2019+368	120	1.8	7.25	60.4	5040	10.1	0.17	0.14
Combined	285	-	1.29	4.51	158	-	2.22	0.8
Crab (HEGRA) 2017 [12]	~ 56	-	-	667	127551	-	.015	.028
Tevatron 2016 [13]	0.442	-	6×10^5	-	-	-	-	-
Crab (HEGRA) 2013 [27]	56	-	40	-	-	-	-	-
RX J1713.7–3946 (HESS) 2008 [15]	30	-	180	-	-	-	-	-
Crab (Themistocle) 1997 [14]	20	-	300	-	-	-	-	-
GRB09510 (*Fermi*-LAT) 2013 $v > c$ [16]	-	-	-	746	123456790	-	0.0134	0.0009
GRB09510 (*Fermi*-LAT) 2013 $v < c$ [16]	-	-	-	1075	59171598	-	0.0093	0.0013
Crab (HEGRA) 2019 [17]	75	2	-	-	-	59	-	-

Derived **95%** CL lower limits on Ec and its different LIV coefficients

HAWC Collaboration
Phys Rev Lett. 124,131101 (2020)

LIV limits

High Altitude Water Cherenkov
Gamma-Ray Observatory

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Crab (Themistocle) 1997 [14]	20	-	300	-	-	-	-	-
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Derived **95%** CL lower limits on Ec and its different LIV coefficients

ASTROPHYSICS AND COSMOLOGY | NEWS

100 TeV photons test Lorentz invariance

2 June 2020

https://cerncourier.com/a/100-tev-photons-test-lorentz-invariance/

Extreme Experiment on Mexican Volcano Challenged the Speed of Light

By Ryan F. Mandelbaum | 4/02/20 5:01PM | Comments (13)

https://gizmodo.com/extreme-experiment-on-mexican-volcano-challenged-the-sp-1842648310

Forbes

Astrophysics Signal Does What The LHC Cannot: Constrain Quantum Gravity And String Theory

Ethan Siegel Senior Contributor
Starts With A Bang Contributor Group
Science
The Universe is out there, waiting for you to discover it.

HAWC Collaboration
Phys Rev Lett. 124,131101 (2020)

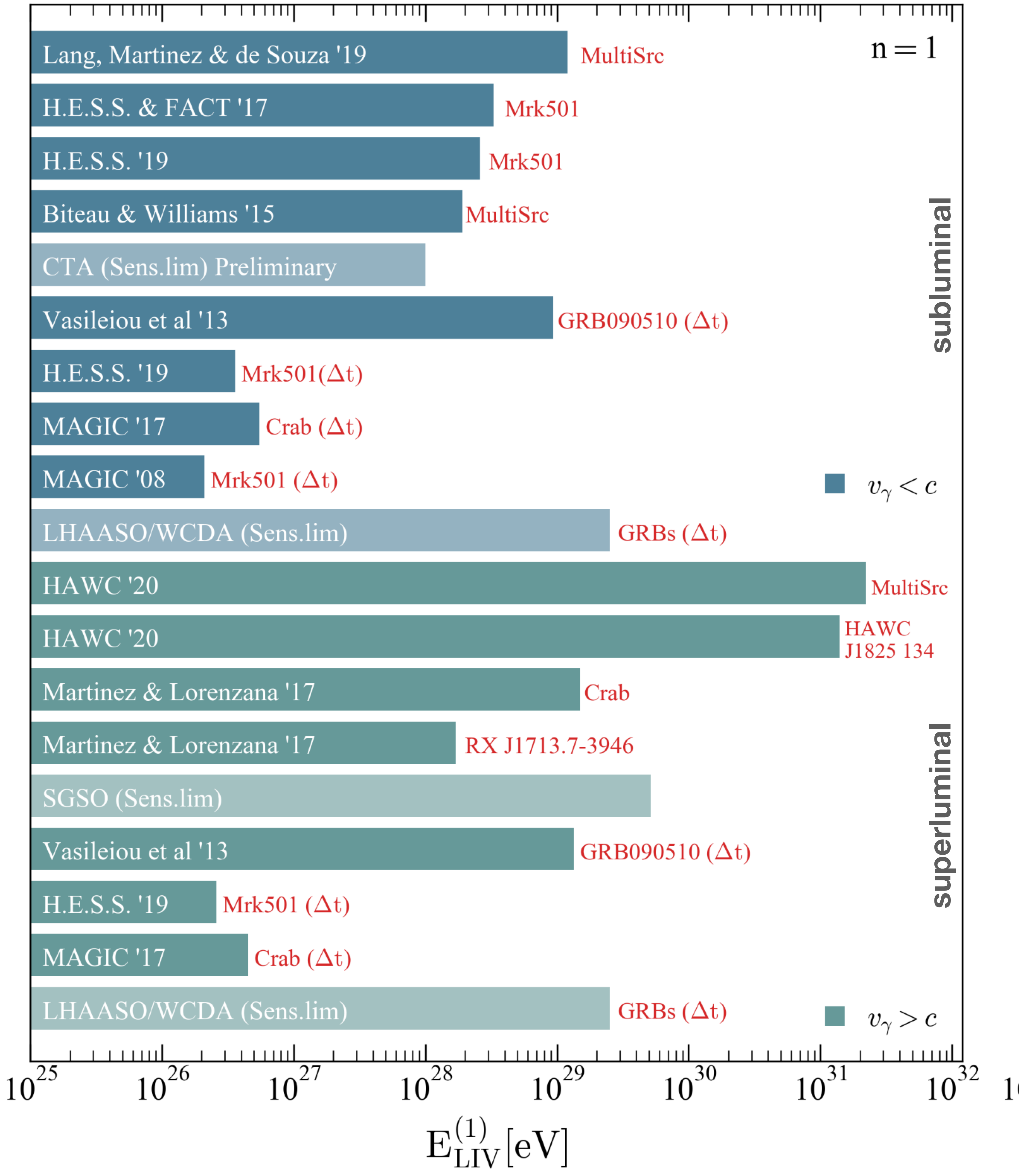
LIV limits

Pair production threshold shifts

Energy-dependent time delay

Photon decay

Energy-dependent time delay

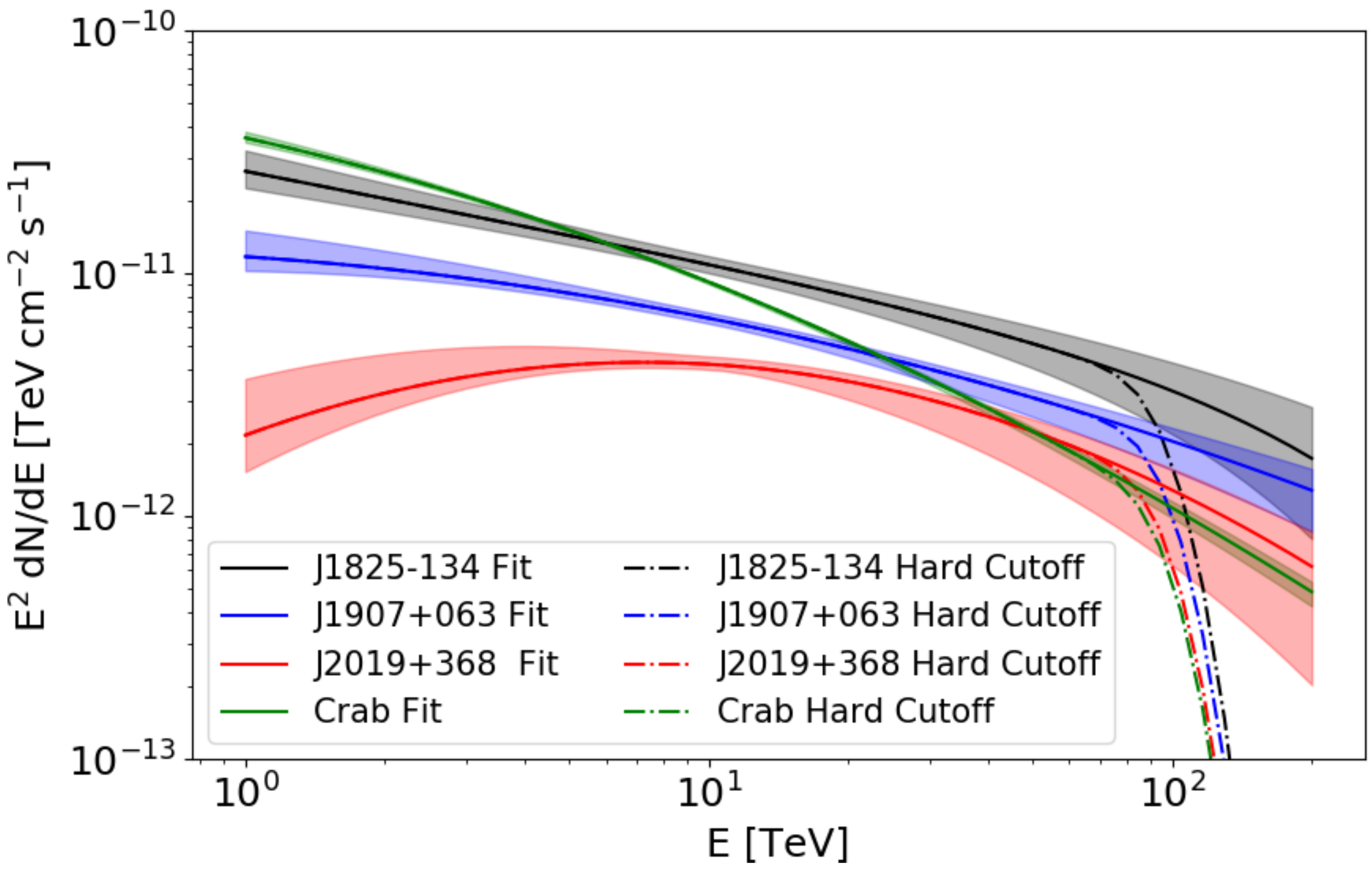


Constraints on Lorentz invariance violation using HAWC observations above 100TeV
HAWC Collaboration,
arXiv:1911.08070
Phys Rev Lett. 124,131101 (2020)

>1800 E_{Pl} !!

High Altitude Water Cherenkov
Gamma-Ray Observatory

LIV hard cutoff



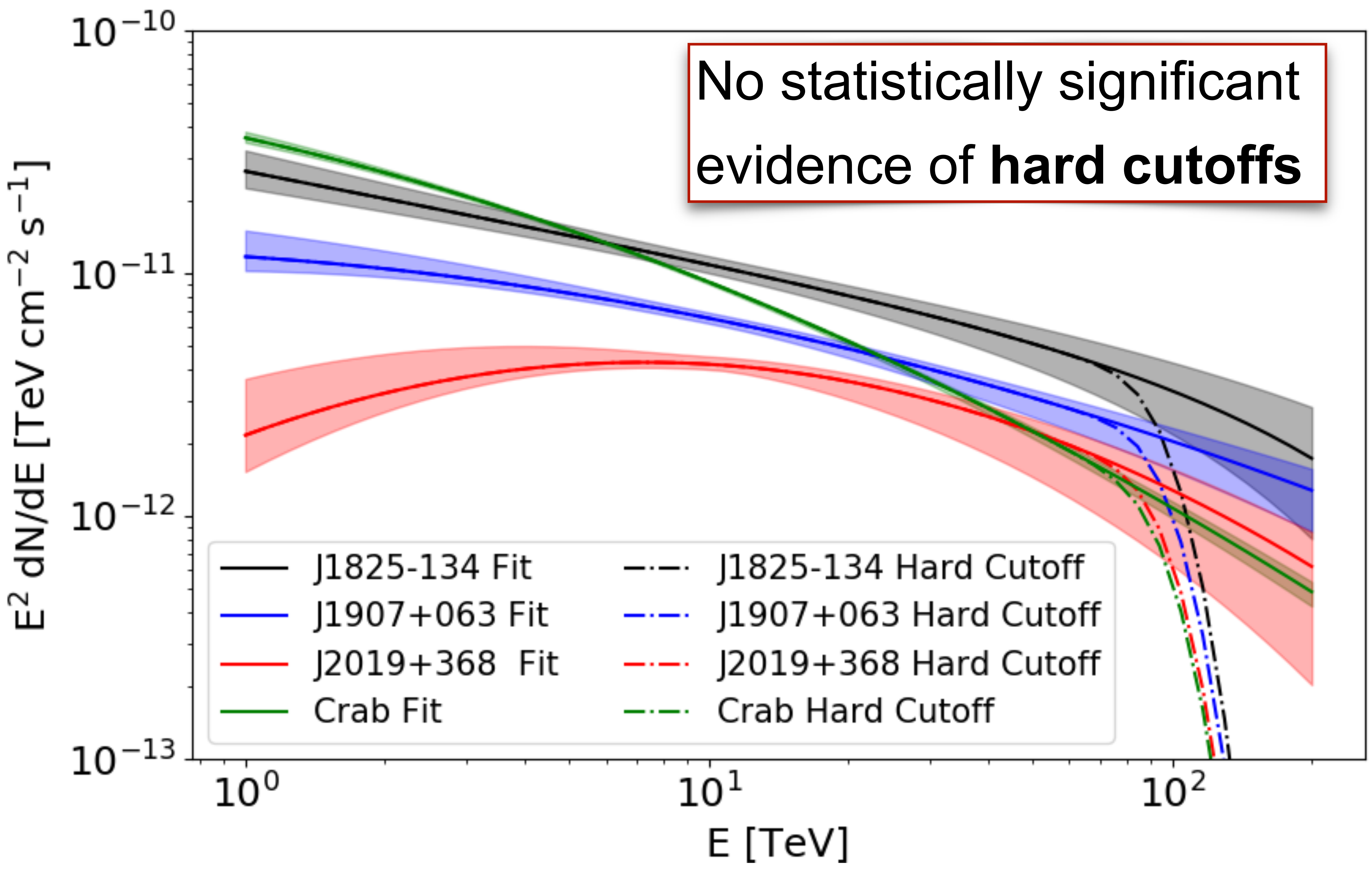
No statistically significant evidence of **hard cutoffs**

Source	E_c TeV	L kpc
J1825-134	244	1.55
J1907+063	218	2.37
J0534+220	152	2
J2019+368	120	1.8
Combined	285	-

humbertomh@ifsc.usp.br

HAWC Collaboration
Phys Rev Lett. 124,131101 (2020)

LIV hard cutoff



$$L\,\Gamma = 1$$

$$E_{LIV}^{(2)} > 3.33 \times 10^{19}\text{eV}\left(\frac{L}{\text{kpc}}\right)^{0.1}\left(\frac{E_\gamma}{\text{TeV}}\right)^{1.9}$$

Source	E_c TeV	L kpc	$E_{LIV}^{(2)}$ (3γ) 10^{23}eV
J1825-134	244	1.55	12
J1907+063	218	2.37	10.1
J0534+220	152	2	4.99
J2019+368	120	1.8	3.15
Combined	285	-	-

humbertomh@ifsc.usp.br

HAWC Collaboration
Phys Rev Lett. 124,131101 (2020)

LIV hard cutoff

Pair production threshold shifts

Energy-dependent time delay

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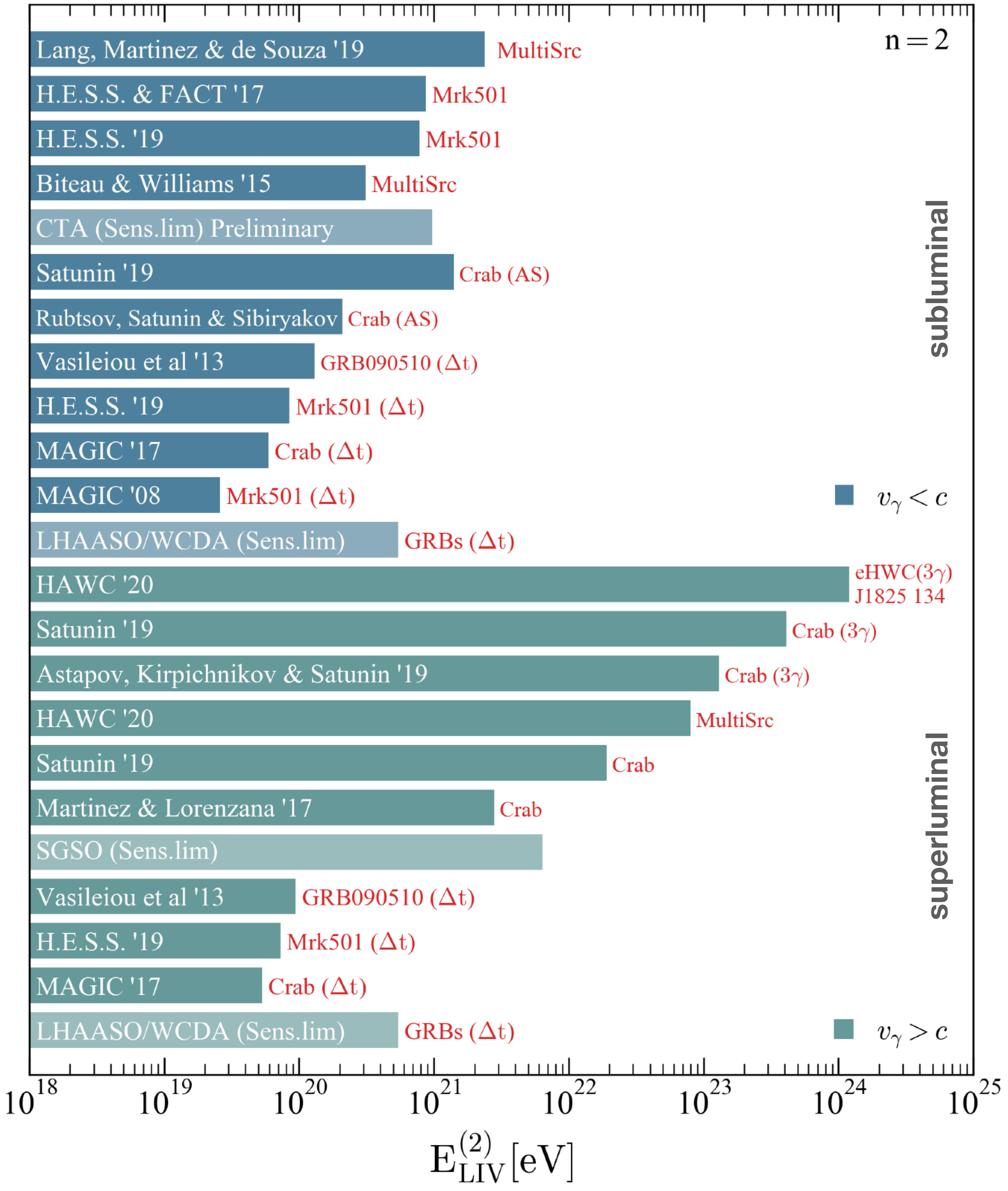
Energy-dependent time delay



Constraints on Lorentz invariance violation using HAWC observations above 100TeV
HAWC Collaboration,
arXiv:1911.08070 (**Submitted**)

Standard Model Extension d=4 (n=0)

Isotropic Lorentz- violating (LV) deformation of the photon sector

$$\mathcal{L}_{modM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\kappa^{\mu\nu}{}_{\rho\lambda}F_{\mu\nu}F^{\rho\lambda}.$$

D. Colladay and V.A. Kostelecký, Phys. Rev. D 58, 116002 (1998)

Kappa:

$$\kappa^{\mu\nu}{}_{\mu\nu} = 0 \quad ; \qquad \kappa_{\mu\nu\rho\lambda} = -\kappa_{\nu\mu\rho\lambda} = \kappa_{\nu\mu\lambda\rho}, \quad \kappa_{\mu\nu\rho\lambda} = \kappa_{\rho\lambda\mu\nu}$$

256 independent components to 19

$$\kappa^{\mu\nu\rho\lambda} = \frac{1}{2}(\eta^{\mu\rho}\tilde{\kappa}^{\nu\lambda} - \eta^{\mu\lambda}\tilde{\kappa}^{\nu\rho} + \eta^{\nu\lambda}\tilde{\kappa}^{\mu\rho} - \eta^{\nu\rho}\tilde{\kappa}^{\mu\lambda}) \; ; \quad \tilde{\kappa}^{\mu\nu} = \frac{3}{2}\tilde{\kappa}_{tr}\ \mathrm{diag}(1, 1/3, 1/3, 1/3)$$

19 independent components to 1

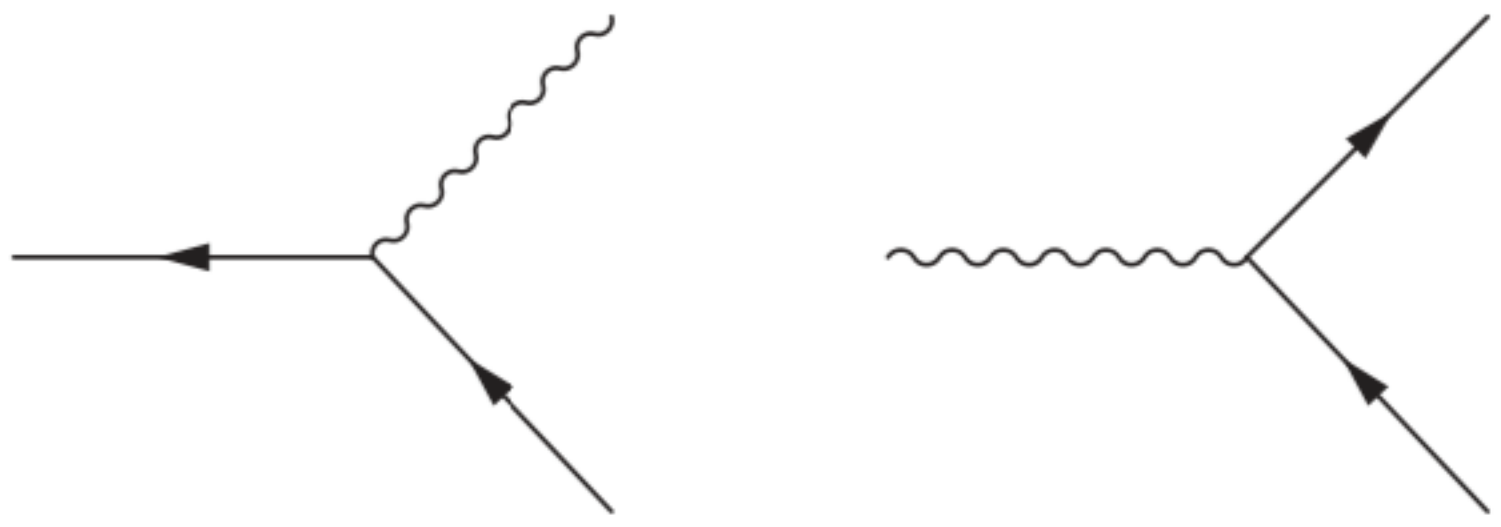
F. R. Klinkhamer and M. Schreck Phys. Rev. D **78**, 085026 (2008)

M. Hohensee, R. Lehnert, D. Phillips, R. Walsworth Phys. Rev. D **80**, 036010 (2009)

LORENTZ-NONINVARIANT DECAY PROCESSES

a) Vacuum Cherenkov radiation

b) Photon decay



(a) (b)

Photon decay Threshold

$$-2\tilde{\kappa}_{tr} \sim 4m_e^2/E_\gamma^2$$

$$\alpha_0 = -2\tilde{\kappa}_{tr}$$

Standard Model Extension d=6 (n=2)

The photon sector of the minimal SME

$$\mathcal{L} = -\tfrac{1}{4}F^2 - \tfrac{1}{4}(k_F)^{\kappa\lambda\mu\nu}F_{\kappa\lambda}F_{\mu\nu} + (k_{AF})^{\mu}A^{\nu}\tilde{F}_{\mu\nu}$$

D. Colladay and V.A. Kostelecký, Phys. Rev. D 58, 116002 (1998)

Dispersion relation

$$E(p) \simeq \left(1 - \varsigma^0 \pm \sqrt{(\varsigma^1)^2 + (\varsigma^2)^2 + (\varsigma^3)^2}\right)p$$

V.A. Kostelecký and M. Mewes, Phys. Rev. D 80, 015020 (2009)

An expansion in mass dimension and spherical decomposition

$$\varsigma^0 = \sum_{djm} p^{d-4} Y_{jm}(\theta_k, \varphi_k) c^{(d)}_{(I)jm}$$

d=6 → $j, m = 0$ **Directional independent**

$$= p^2 \sqrt{\frac{1}{4\pi}} c^{(6)}_{(I)00}$$

$$\varsigma^{\pm} = \varsigma^1 \pm \varsigma^2$$

$$= \sum_{djm} p^{d-4} {}_{\mp 2}Y_{jm}(\theta_k, \varphi_k)(k^{(d)}_{(E)jm} \mp i k^{(d)}_{(B)jm}),$$

$$\varsigma^3 = \sum_{djm} p^{d-4} Y_{jm}(\theta_k, \varphi_k) k^{(d)}_{(V)jm},$$

$$= 0$$

$$-\alpha_2 = c^{(6)}_{(I)00}/\sqrt{\pi}$$

Standard Model Extension d=6 (n=2)

Directional dependent $j, m \neq 0$

$$\varsigma^0 = \sum_{djm} p^{d-4} Y_{jm}(\theta_k, \varphi_k) c^{(d)}_{(I)jm}$$

...frames centered on the Sun, the galaxy, and the CMB each remain unchanged approximate inertial frames over thousands of years

R. Bluhm et al.,
Phys. Rev. Lett. 88, 090801 (2002)

Sec. V: V.A. Kostelecký and M. Mewes,
Phys. Rev. D 80, 015020 (2009)

Sun centered celestial equatorial frame

$$\theta_k = 90^o - \mathrm{Dec}(^o)$$

$$\varphi_k = \mathrm{RA}(^o)$$

F. Kislat and H. Krawczynsky
Phys. Rev. D 92, 045016 (2015)
V. Vasileiou et al.,
Phys. Rev. D 87, 122001 (2013)

$$-\alpha_2 = 2 \sum_{6jm} Y_{jm}(\theta_k, \varphi_k) c^{(6)}_{(I)jm}$$

This Work	**eHWC J1825-134:**	$\sum_{jm} Y_{jm}(103.37^o, 276.40^o) c^{(6)}_{(I)jm}$	$-1.3 \times 10^{-28} \mathrm{GeV}^{-2}$	**Limits!**
CPT Tables D18 p.49 ArXiv 0801.0287v12-2	**GRB 90510:**	$\sum_{jm} Y_{jm}(116^\circ, 334^\circ) c^{(6)}_{(I)jm}$	$1.4 \times 10^{-21}\ \mathrm{GeV}^{-2}$ $-0.31 \times 10^{-20} \mathrm{GeV}^{-2}$	(V. Vasileiou et al., 2013)

arXiv.org > hep-ph > arXiv:0801.0287

High Energy Physics – Phenomenology

[Submitted on 1 Jan 2008 (v1), last revised 2 Jan 2021 (this version, v14)]

Data Tables for Lorentz and CPT Violation

Alan Kostelecky, Neil Russell

Submission history

From: Alan Kostelecky [view email]

[v1] Tue, 1 Jan 2008 09:41:36 UTC (10 KB)
[v2] Thu, 22 Jan 2009 18:29:13 UTC (78 KB)
[v3] Tue, 5 Jan 2010 02:00:01 UTC (82 KB)
[v4] Thu, 6 Jan 2011 21:28:27 UTC (74 KB)
[v5] Fri, 13 Jan 2012 11:38:36 UTC (85 KB)
[v6] Thu, 24 Jan 2013 01:34:25 UTC (90 KB)
[v7] Thu, 23 Jan 2014 00:07:28 UTC (98 KB)
[v8] Mon, 19 Jan 2015 23:41:30 UTC (101 KB)
[v9] Fri, 26 Feb 2016 20:07:41 UTC (105 KB)
[v10] Fri, 13 Jan 2017 16:04:18 UTC (113 KB)
[v11] Mon, 8 Jan 2018 20:45:38 UTC (118 KB)
[v12] Thu, 3 Jan 2019 17:03:59 UTC (123 KB)
[v13] Fri, 3 Jan 2020 19:20:27 UTC (126 KB)
[v14] Sat, 2 Jan 2021 02:35:44 UTC (130 KB)

Data Tables for Lorentz and CPT Violation

V. Alan Kostelecký[a] and Neil Russell[b]
[a] *Physics Department, Indiana University, Bloomington, IN 47405*
[b] *Physics Department, Northern Michigan University, Marquette, MI 49855*

January 2021 update of *Reviews of Modern Physics* **83**, 11 (2011) [arXiv:0801.0287]

This work tabulates measured and derived values of coefficients for Lorentz and CPT violation in the Standard-Model Extension. Summary tables are extracted listing maximal attained sensitivities in the matter, photon, neutrino, and gravity sectors. Tables presenting definitions and properties are also compiled.

CONTENTS

The Lorentz-violating operators in the SME are systematically classified according to their mass dimension, and operators of arbitrarily large dimension can appear. At any fixed dimension, the operators are finite in number and can in principle be enumerated. A limiting case of particular interest is the minimal SME, which can be

Table D17. Nonminimal photon

Combination	Result		
$\lvert k^{(5)}_{(V)00}\rvert$	$< 3.5 \times 10^{-2}$		
$\lvert k^{(5)}_{(V)10}\rvert$	$< 4.0 \times 10^{-2}$		
$\lvert \mathrm{Re}\, k^{(5)}_{(V)11}\rvert$	$< 2.3 \times 10^{-2}$		
$\lvert \mathrm{Im}\, k^{(5)}_{(V)11}\rvert$	$< 2.2 \times 10^{-2}$		
$\lvert k^{(5)}_{(V)20}\rvert$	$< 3.6 \times 10^{-2}$		
$\lvert \mathrm{Re}\, k^{(5)}_{(V)21}\rvert$	$< 3.0 \times 10^{-2}$		
$\lvert \mathrm{Im}\, k^{(5)}_{(V)21}\rvert$	$< 3.0 \times 10^{-2}$		
$\lvert \mathrm{Re}\, k^{(5)}_{(V)22}\rvert$	$< 1.6 \times 10^{-2}$		
$\lvert \mathrm{Im}\, k^{(5)}_{(V)22}\rvert$	$< 1.5 \times 10^{-2}$		
$\lvert k^{(5)}_{(V)30}\rvert$	$< 2.7 \times 10^{-2}$		
$\lvert \mathrm{Re}\, k^{(5)}_{(V)31}\rvert$	$< 2.8 \times 10^{-2}$		
$\lvert \mathrm{Im}\, k^{(5)}_{(V)31}\rvert$	$< 2.7 \times 10^{-25}$ GeV^{-1}	”	
$\lvert \mathrm{Re}\, k^{(5)}_{(V)32}\rvert$	$< 2.5 \times 10^{-25}$ GeV^{-1}	”	
$\lvert \mathrm{Im}\, k^{(5)}_{(V)32}\rvert$	$< 2.0 \times 10^{-25}$ GeV^{-1}	”	
$\lvert \mathrm{Re}\, k^{(5)}_{(V)33}\rvert$	$< 1.8 \times 10^{-25}$ GeV^{-1}	”	
$\lvert \mathrm{Im}\, k^{(5)}_{(V)33}\rvert$	$< 1.6 \times 10^{-25}$ GeV^{-1}	”	
$k^{(5)}_{(V)00}$	$< 6.86 \times 10^{-20}$ GeV^{-1}	Astrophysics	[162]

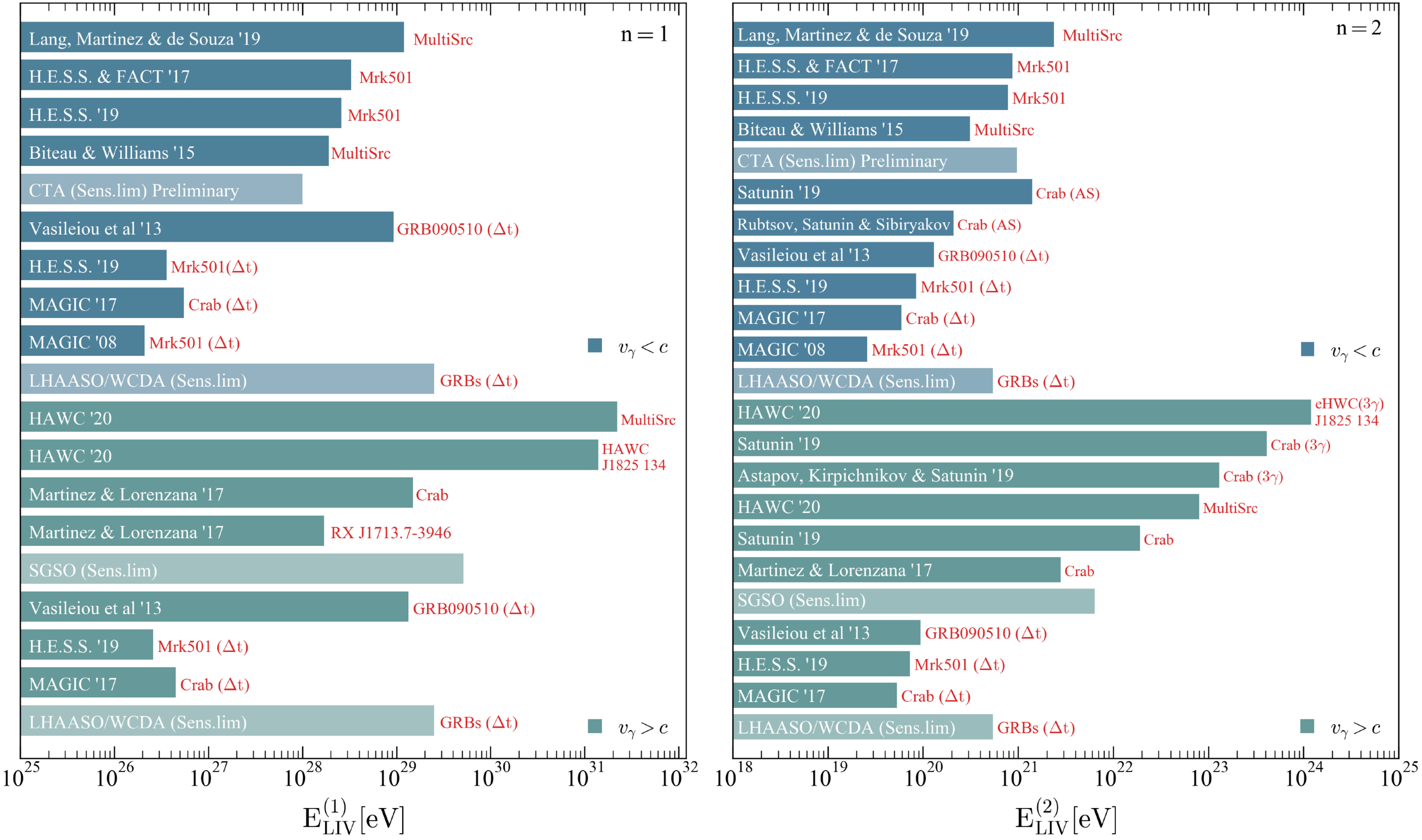
Table D16. Photon sector, $d = 4$ (part 6 of 7)

Combination	Result	System	Ref.
$\lvert c^{(4)}_{(I)00}\rvert$	$< 2.28 \times 10^{-8}$	Astrophysics	[162]
$\lvert \sum_{jm} Y_{jm}(103.45°, 276.41°) c^{(4)}_{(I)jm}\rvert$	$< 8.77 \times 10^{-9}$	”	[162]
$\lvert \sum_{jm} Y_{jm}(83.75°, 286.95°) c^{(4)}_{(I)jm}\rvert$	$< 11.5 \times 10^{-9}$	”	[162]
$\lvert \sum_{jm} Y_{jm}(67.96°, 83.6°) c^{(4)}_{(I)jm}\rvert$	$< 22.6 \times 10^{-9}$	”	[162]
$\lvert \sum_{jm} Y_{jm}(53.26°, 304.94°) c^{(4)}_{(I)jm}\rvert$	$< 36.3 \times 10^{-9}$	”	[162]
$\lvert c^{(4)}_{(I)00}\rvert$	$< 3.3 \times 10^{-9}$	Laser interferometry	[141]
$\lvert c^{(4)}_{(I)10}\rvert$			
$\lvert \mathrm{Re}\, c^{(4)}_{(I)11}\rvert$			
$\lvert \mathrm{Im}\, c^{(4)}_{(I)11}\rvert$			
$\lvert c^{(4)}_{(I)21}\rvert$			
$\lvert c^{(4)}_{(I)22}\rvert$			
$c^{(4)}_{(I)10}$			
”			

Table D16. Photon sector, $d = 4$ (part 5 of 7)

Combination	Result	System	Ref.
$\lvert\tilde{\kappa}_{\mathrm{tr}}\rvert$	$< 6.43 \times 10^{-18}$	Astrophysics	[162]
$\tilde{\kappa}_{\mathrm{tr}}$	$> -3 \times 10^{-19}$	”	[163]*
$\lvert\tilde{\kappa}_{\mathrm{tr}}\rvert$	$< 9.2 \times 10^{-10}$	Laser interferometry	[141]
$\tilde{\kappa}_{\mathrm{tr}}$	$(-6.0 \pm 4.0) \times 10^{-10}$	Sapphire cavity oscillators	[143]
$\lvert\tilde{\kappa}_{\mathrm{tr}}\rvert$	$< 2 \times 10^{-8}$	Relativistic Li ions	[65]
$\tilde{\kappa}_{\mathrm{tr}}$	$(-2 \text{ to } 0.0006) \times 10^{-16}$	Astrophysics	[164]*
”	$(-0.4 \pm 0.9) \times 10^{-10}$	Optical ring cavity	[156]
”	$(3 \pm 11) \times 10^{-10}$	Asymmetric optical resonator	[157]
”	$(3.4 \pm 6.2) \times 10^{-9}$	”	[158]
”	$(-1.5 \pm 0.74) \times 10^{-8}$	Rotating microwave resonators	[144]
”	$(-0.3 \pm 3) \times 10^{-7}$	Microwave interferometer	[165]

Strong LIV Exclusion limits in the photon sector by astroparticle tests



H. Martínez-Huerta, R. G. Lang and V. de Souza.
Symmetry 2020, 12, 1232.

Conclusions and remarks

- **Astroparticle physics has recently reached a new status of precision** due to the construction of new observatories, operating innovative technologies, and the detection of large numbers of events and sources.

  ➡ The precise measurements of cosmic and gamma rays can be used as tests for fundamental physics, such as effects motivated by some Lorentz invariance violation.

- There are different types of astrophysical LIV predictions through the generic modification to particle dispersion relation in the photon sector, such as **pair production threshold shifts, energy-dependent time delay, photon splitting, and photon decay**.

- **So far, there hasn't been found any confirmed signature of any LIV**

  ➡ **There is an active and dynamic field in astroparticle physics looking for LV/LIV signatures.**

  - There are studies in progress to study the potential to test / constrain LIV signatures in astroparticle physics Experiments: **HAWC, Auger, Magic, Veritas, HESS, SGSO, CTA...**

Exercises

- 1. **What is the minimum energy that a background photon must have to interacts with a 50 TeV gamma ray to produce an e+ e- pair?**
- **2. What is the minimum energy that a background photon must have to interacts with a 50 TeV gamma ray to produce an e+ e- pair?**
- **3. Using ebltable compare the energy densities, optical depth, and attenuation for at least 3 different models and different values for z.**
  - **Compare your results including LIV attenuation (E_LIV= Mpl and n=1)**
    - ***Compare for different values**
- **4. What would the attenuation look like with LIV and without for a source at z= 0.034**

$$\phi_{\rm int}(E_\gamma) = \phi_0(E_\gamma/E_0)^{-\Gamma\cdot}\,\exp\left(-E_\gamma/E_{\rm cut}\right),$$

$$\phi_{\rm int}(E_\gamma) = \phi_0(E_\gamma/E_0)^{-\Gamma\cdot}\,\exp\left(-E_\gamma/E_{\rm cut}\right),$$

M.-H. Ulrich, et al. ApJ 198, 261–266

E_0 [TeV]	Normalization [/cm^2 s TeV]	Γ
1.42	8.27×10^{-12}	2.19

$E_{cut} = 40 TeV$

$E_{cut} = 60 TeV$

4*: Use the LIV attenuation from ebltable in a gammapy analysis

- **5. Modify the ebl_from_model to use (+) scenario**

Drive

IFSC - LIV 2024

Descargar todo

Nombre ↑	Propietario	Última ...
example_0.ipynb	Se ocultó el prc	22 feb 2024
Example_Attenuation.ipynb	Se ocultó el prc	11:58 a.m.

- 6. Find the dispersion relations for

a) $$\mathcal{L} = -\frac{1}{2}\left(\frac{\partial\phi}{\partial x_\mu}\frac{\partial\phi}{\partial x_\mu} + m^2\phi^2\right)$$

b) $$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

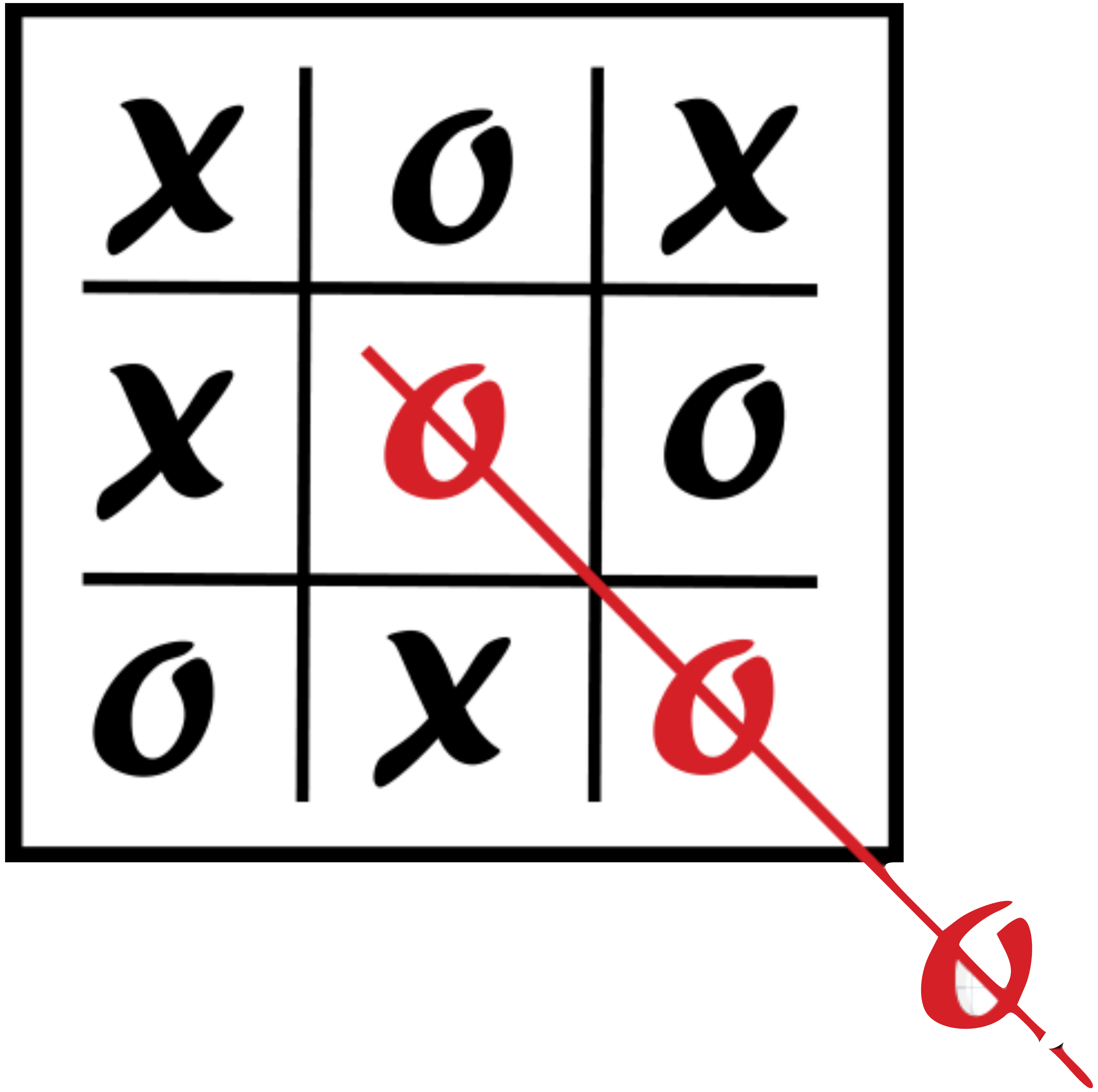
c) $$\mathcal{L} = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} \pm \epsilon F_{\mu\nu}F^{\mu\nu}$$

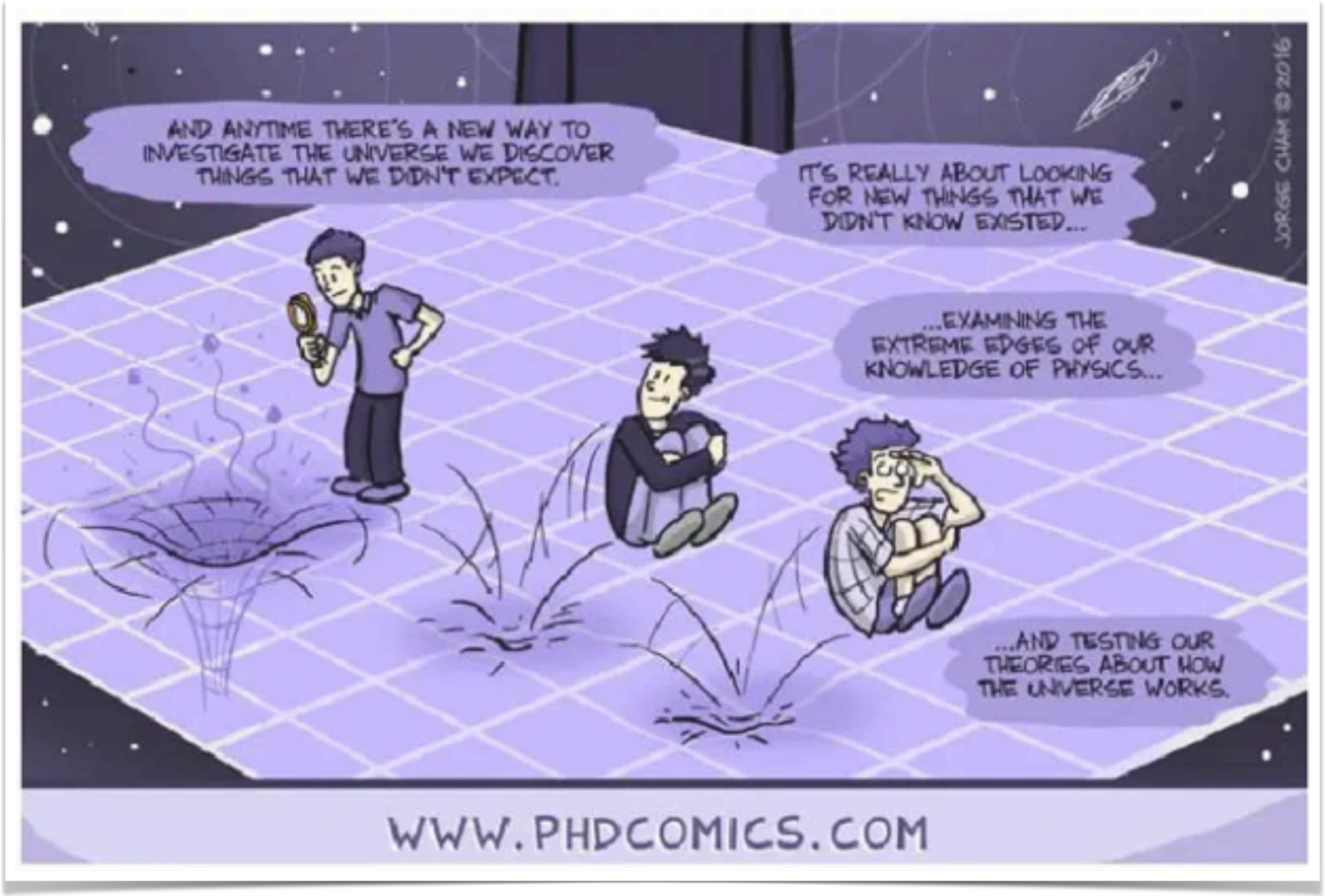
d) $$\mathcal{L} = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} \pm \epsilon F_{\mu\nu}F^{\mu\nu}$$

$$\alpha = \left(1 \quad \pm\epsilon\right)$$

i) $$\partial_\mu \alpha = 0$$

ii) $$\partial_\mu \alpha = B_\mu$$





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INSPIRANDO TU MEJOR VERSIÓN **UDEM**

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Thanks!